

La science quantique

Une vision singulière

X) Qubits optiques A

P.A. Besse

Polarisation et génération de Qubits optiques

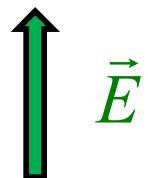
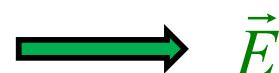
Horizontal / Vertical → Qubit

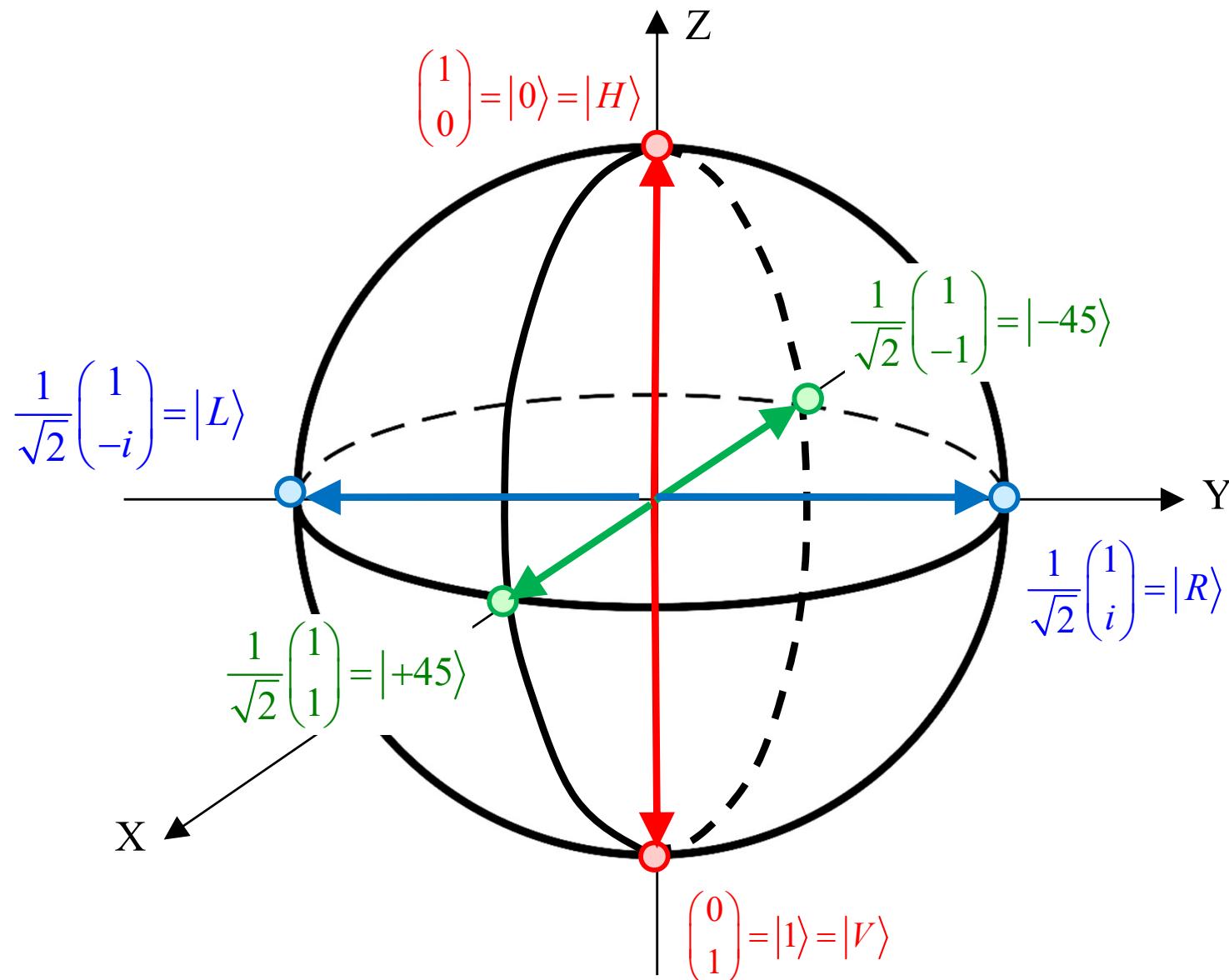
$$|H\rangle = |h\rangle = |0\rangle \quad |V\rangle = |v\rangle = |1\rangle \quad |\psi\rangle = a_0 \cdot |H\rangle + a_1 |V\rangle$$

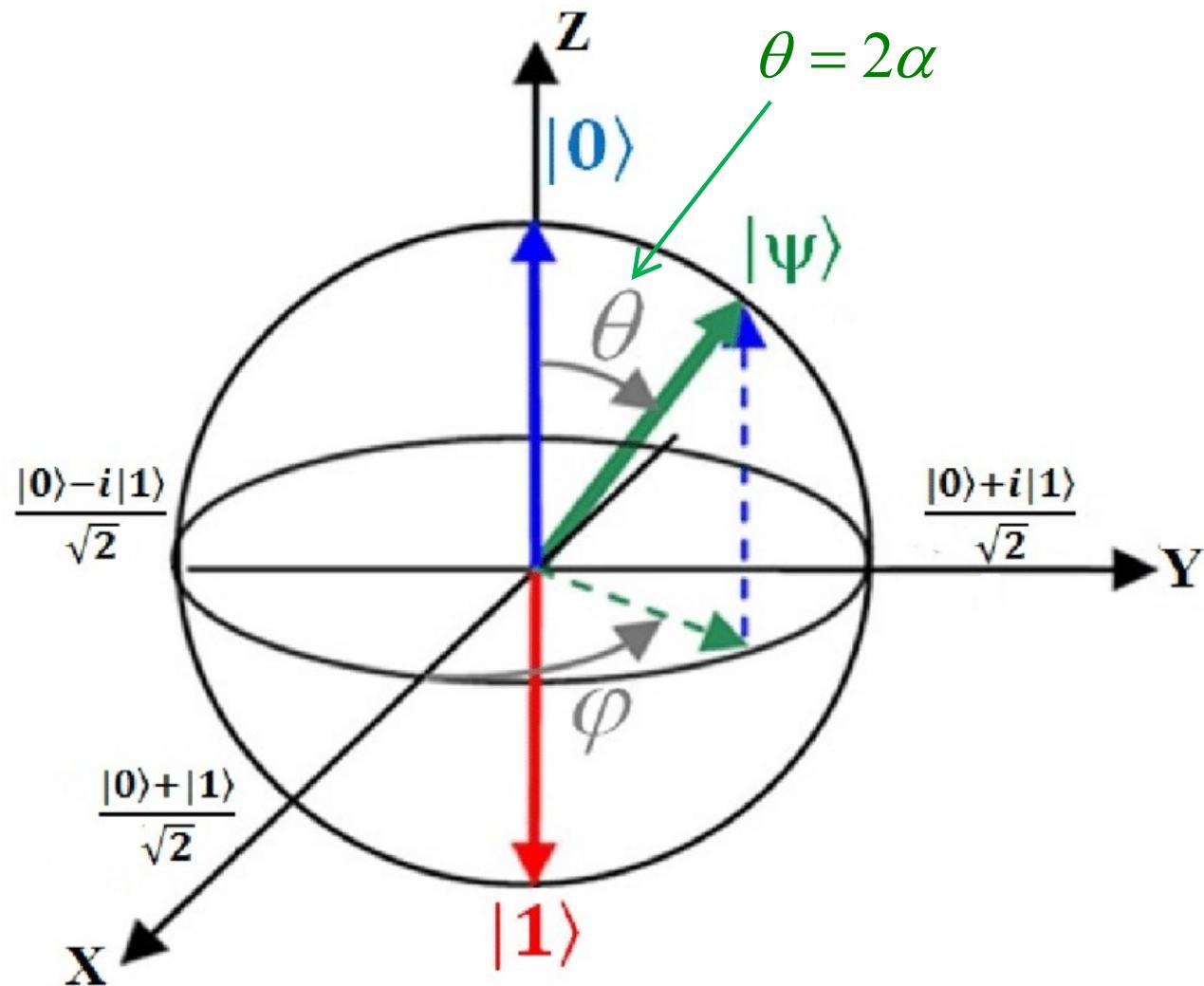
$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

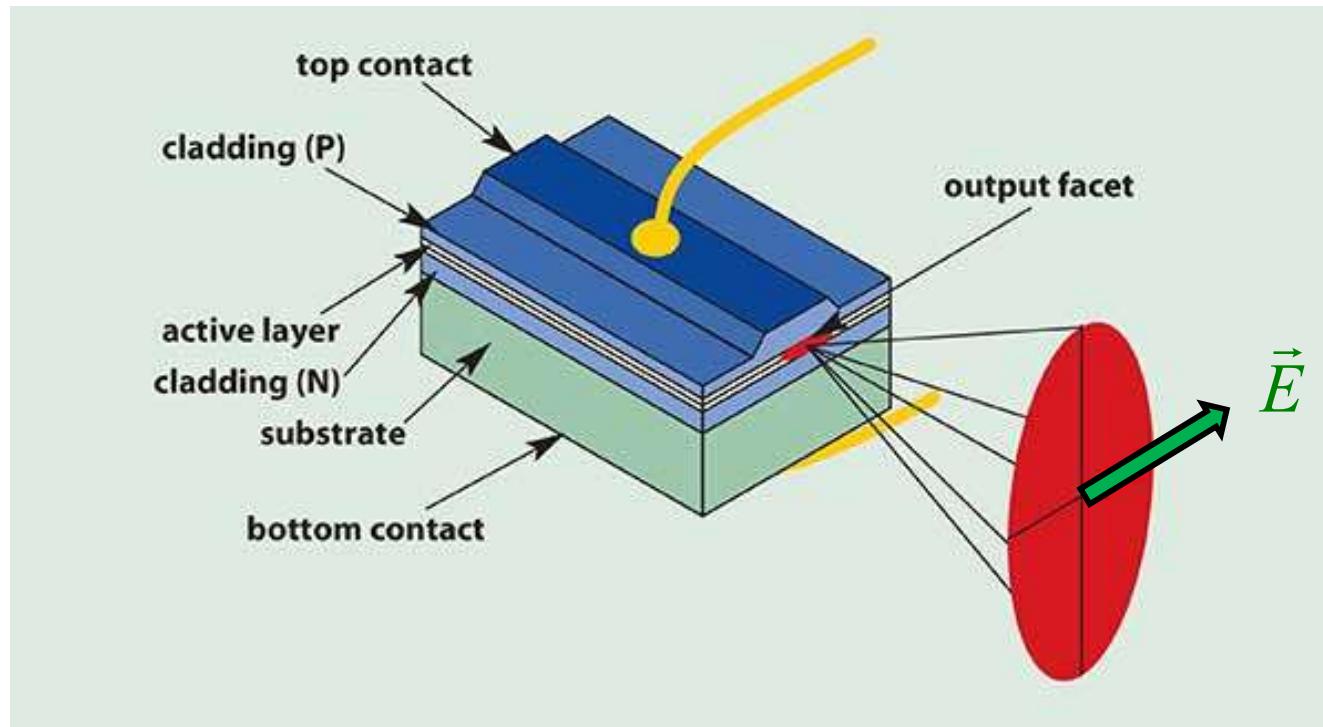
$$\begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$$







$$|\psi\rangle = \begin{pmatrix} \cos \alpha \\ \sin \alpha \cdot e^{i\varphi} \end{pmatrix}$$

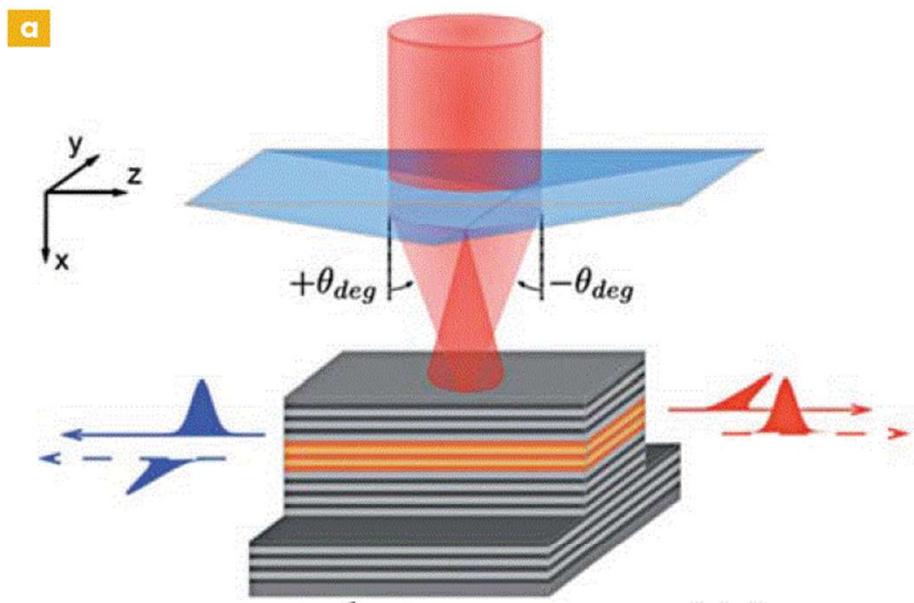


Profile fortement horizontal → Emission polarisée → Qubit de base $|H\rangle$

<https://www.rpmclasers.com/blog/laser-diode-fundamentals-beam-properties/>

Paire de photons intriqués

Phys. Rev. Lett. **110**, 160502 (2013)



$$|\psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle|\rightarrow\rangle + |\rightarrow\rangle|\uparrow\rangle)$$

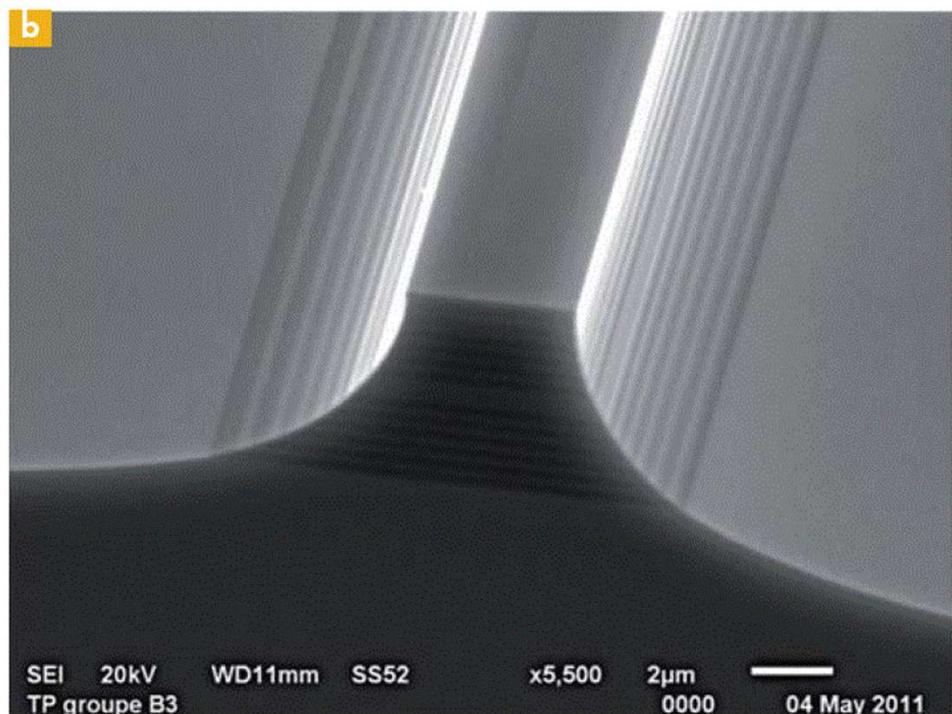
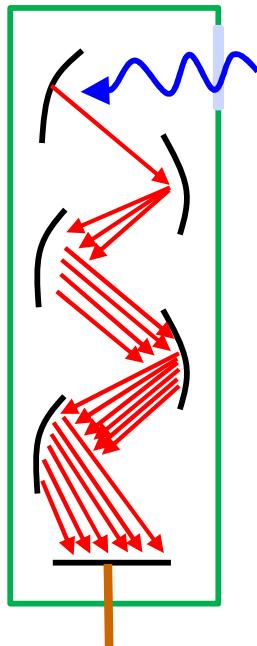


Figure 3. (a) Schéma d'une microcavité semi-conductrice pour la génération de paires de photons intriqués contra-propageants : le faisceau de pompe est transverse par rapport aux photons guidés dans le dispositif. **(b)** Photographie au microscope électronique à balayage du guide d'onde ; la largeur du ruban est de 4 μm , sa longueur 1 mm.

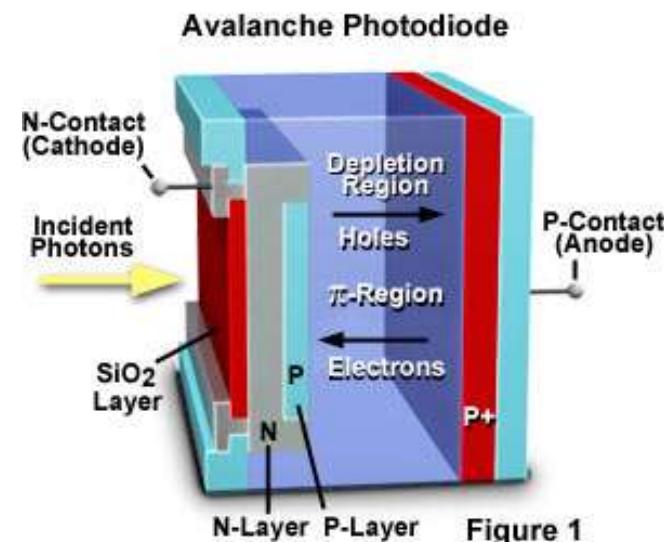
<https://www.photoniques.com/articles/photon/pdf/2014/03/photon201471p24.pdf>

Single Photon Detectors

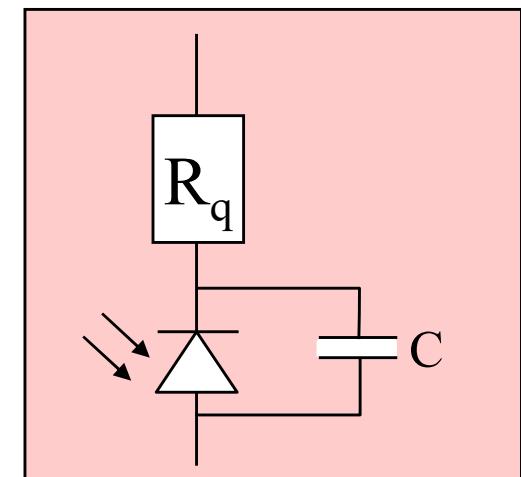
Photo Multiplier Tubes
(PMT)

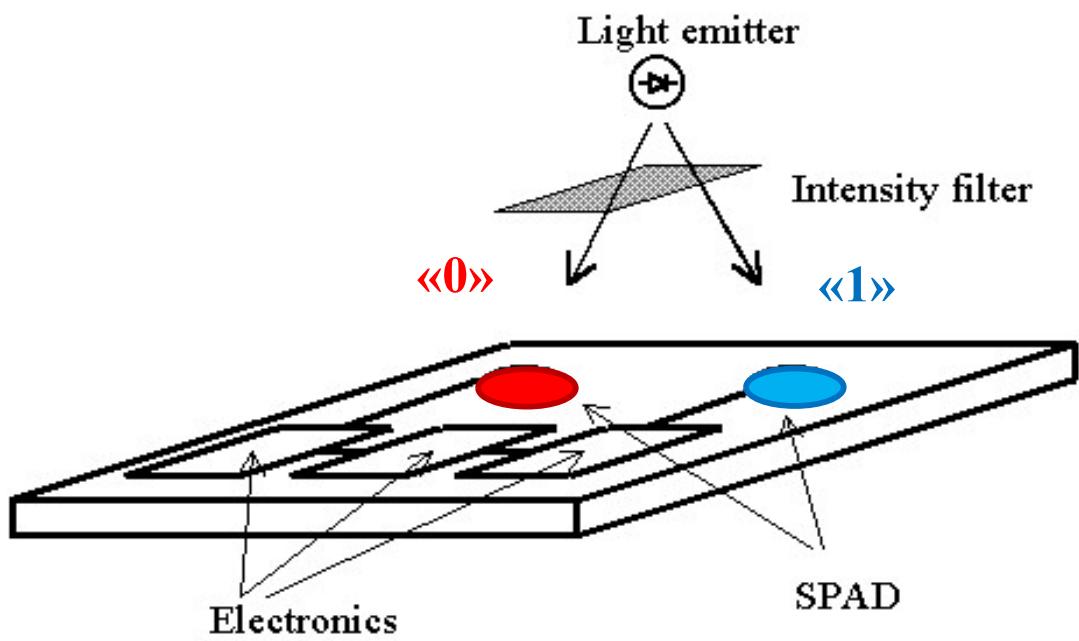


Avalanche Photodiodes
(APD)



Single Photon Avalanche
Diode (SPAD)





CTI project with IdQuantique SA, Genève.

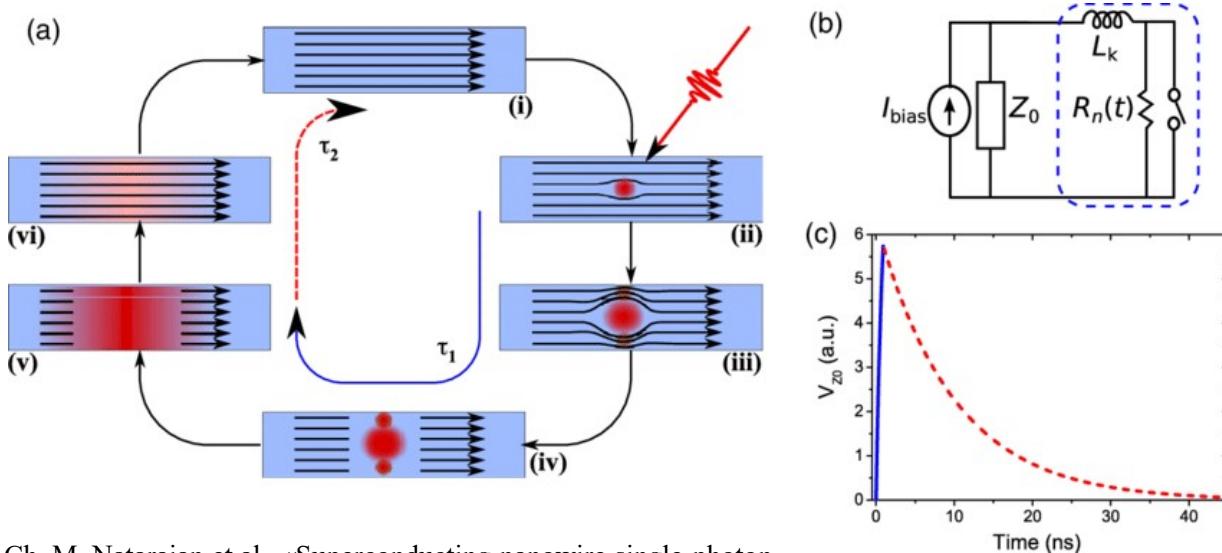


- Data integrity and confidentiality
- User and system authentication
- Security of virtual machines, servers, gateways and remote desktops
- Online Gaming / Random drawings
- Virtual Private Networks encryption
- PKI and certificate management
- Mobile subscribers authentication
- Financial transactions and blockchain
- Randomness-as-a-service (RaaS)
- Simulations and modelling

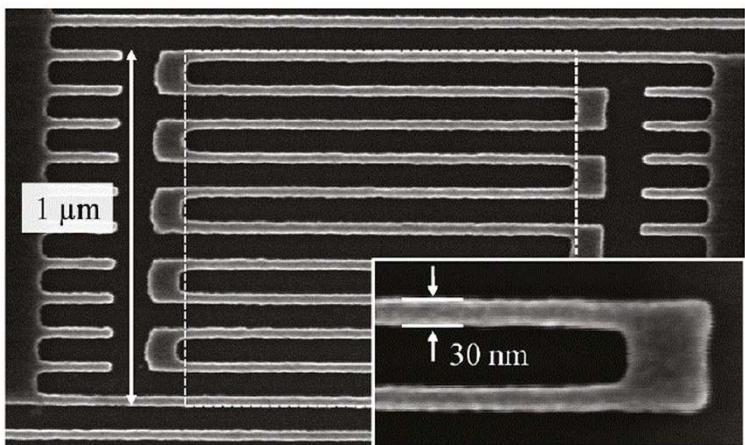
<https://www.idquantique.com/random-number-generation/>

IdQuantique

Superconducting nanowires



Ch. M. Natarajan et al., «Superconducting nanowire single-photon detectors: physics and applications» Supercond. Sci. Technol. 25 (2012) 063001



F. Marsili et al, «Single-Photon Detectors Based on Ultranarrow Superconducting Nanowires», Nano Lett. 2011, 11, 2048–2053

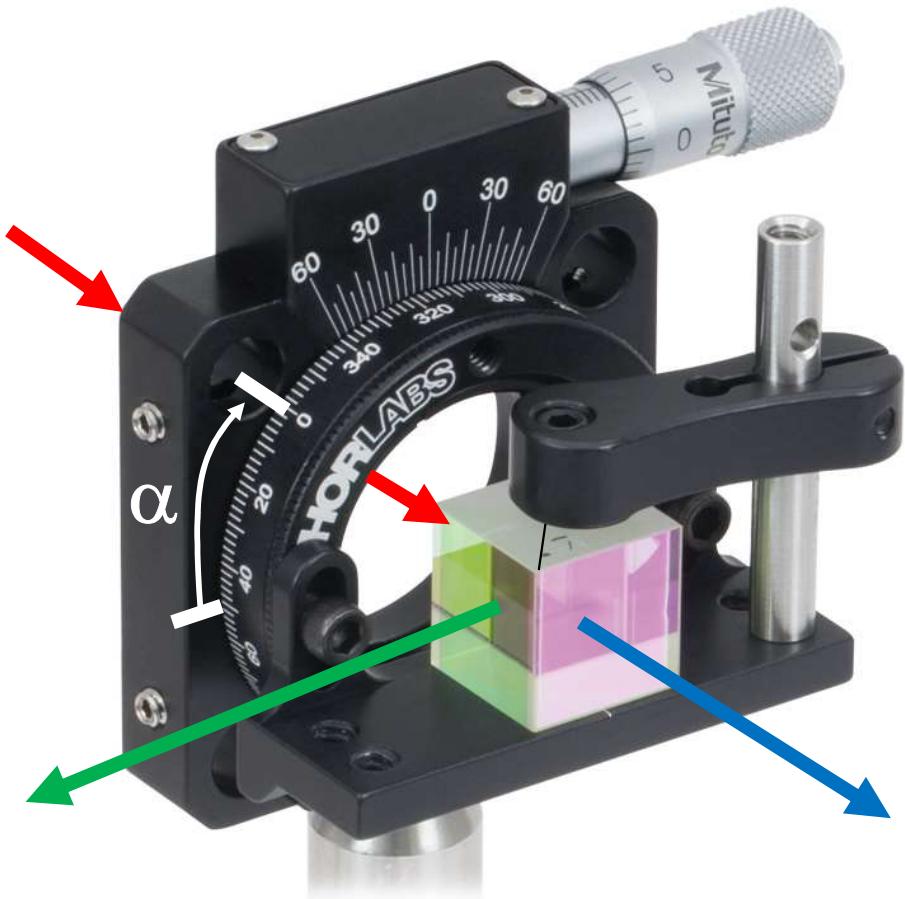
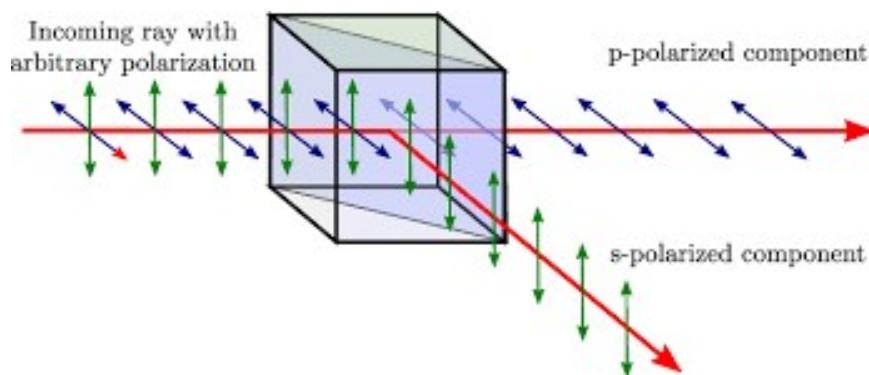
<https://www.idquantique.com/quantum-sensing/products/id281-snspd-series/>



IdQuantique

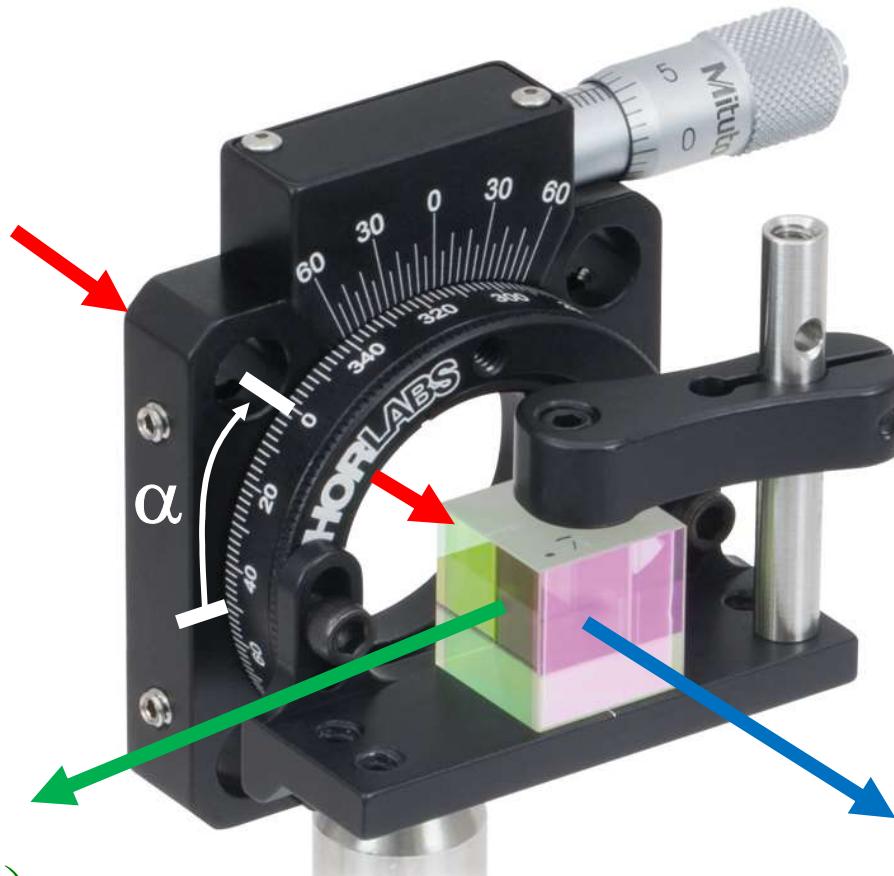
Mesures de polarisation

Polarization Beam Splitter



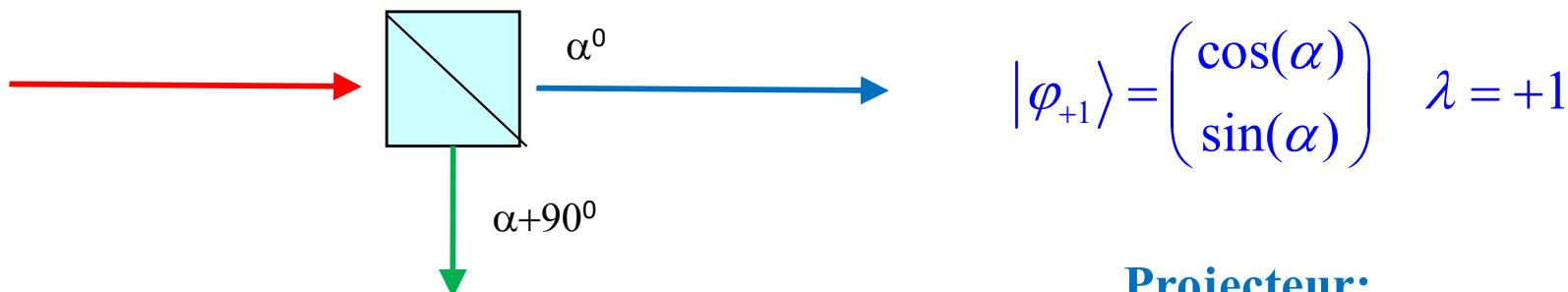
Etats de base du polariseateur

$$\frac{\theta}{2} \equiv \alpha$$



$$|\varphi_{-1}\rangle = \begin{pmatrix} \sin(\alpha) \\ -\cos(\alpha) \end{pmatrix} \quad \lambda = -1$$

$$|\varphi_{+1}\rangle = \begin{pmatrix} \cos(\alpha) \\ \sin(\alpha) \end{pmatrix} \quad \lambda = +1$$



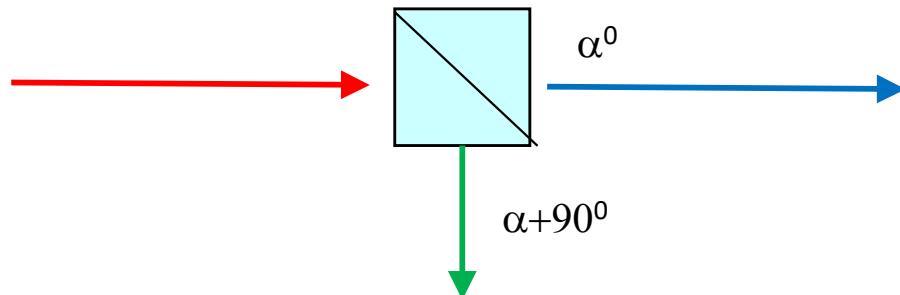
Projecteur:

$$|\varphi_{-1}\rangle = \begin{pmatrix} \sin(\alpha) \\ -\cos(\alpha) \end{pmatrix} \quad \lambda = -1$$

$$P_{+1} = |\varphi_{+1}\rangle \cdot \langle \varphi_{+1}| = \begin{pmatrix} \cos^2(\alpha) & \cos(\alpha)\sin(\alpha) \\ \cos(\alpha)\sin(\alpha) & \sin^2(\alpha) \end{pmatrix}$$

Projecteur:

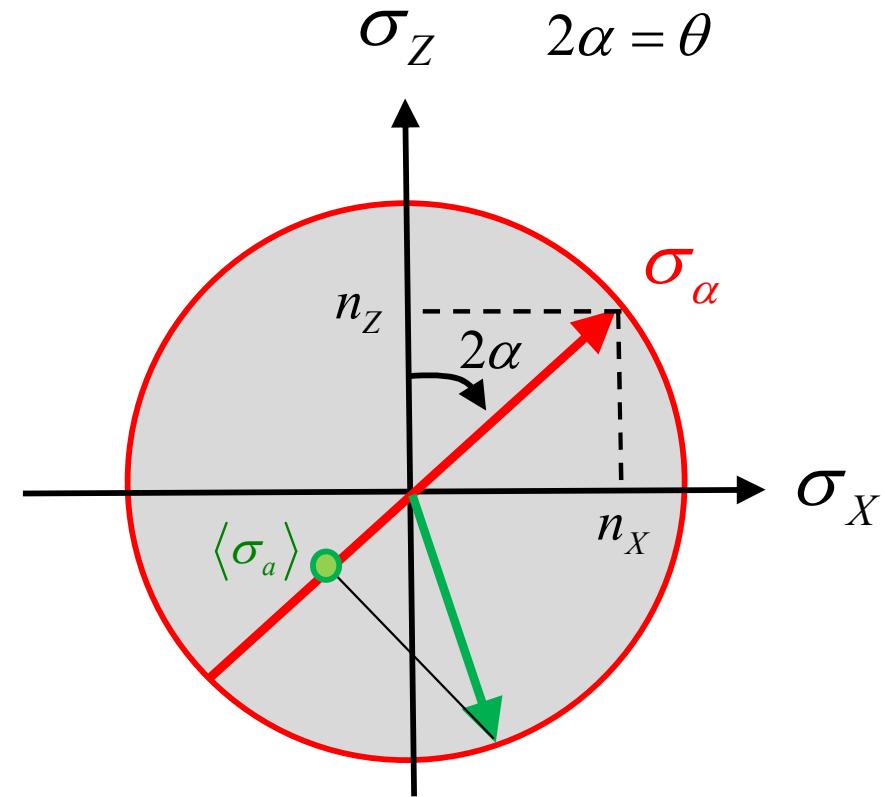
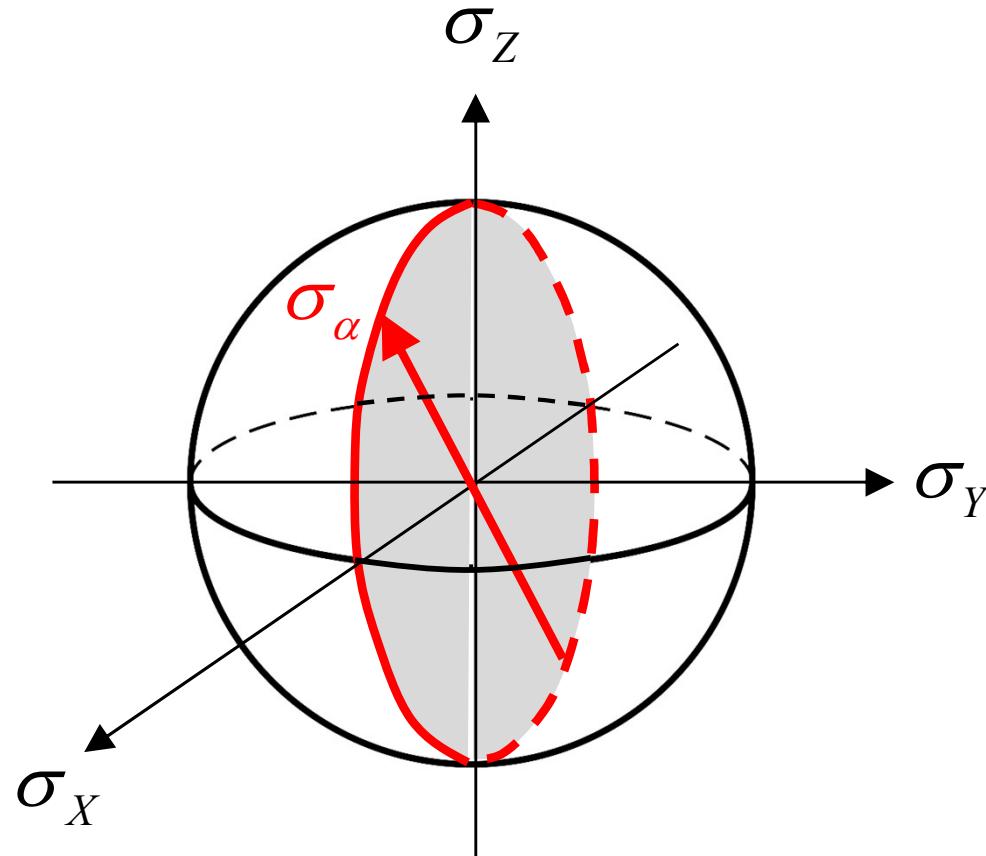
$$P_{-1} = |\varphi_{-1}\rangle \cdot \langle \varphi_{-1}| = \begin{pmatrix} \sin^2(\alpha) & -\cos(\alpha)\sin(\alpha) \\ -\cos(\alpha)\sin(\alpha) & \cos^2(\alpha) \end{pmatrix}$$



$$\sigma_\alpha = \sum_i \lambda_i \cdot P_i = \begin{pmatrix} \cos^2(\alpha) - \sin^2(\alpha) & 2\cos(\alpha)\sin(\alpha) \\ 2\cos(\alpha)\sin(\alpha) & \sin^2(\alpha) - \cos^2(\alpha) \end{pmatrix}$$

$$\sigma_\alpha = \begin{pmatrix} \cos(2\alpha) & \sin(2\alpha) \\ \sin(2\alpha) & -\cos(2\alpha) \end{pmatrix} = \sin(2\alpha) \cdot \sigma_X + \cos(2\alpha) \cdot \sigma_Z$$

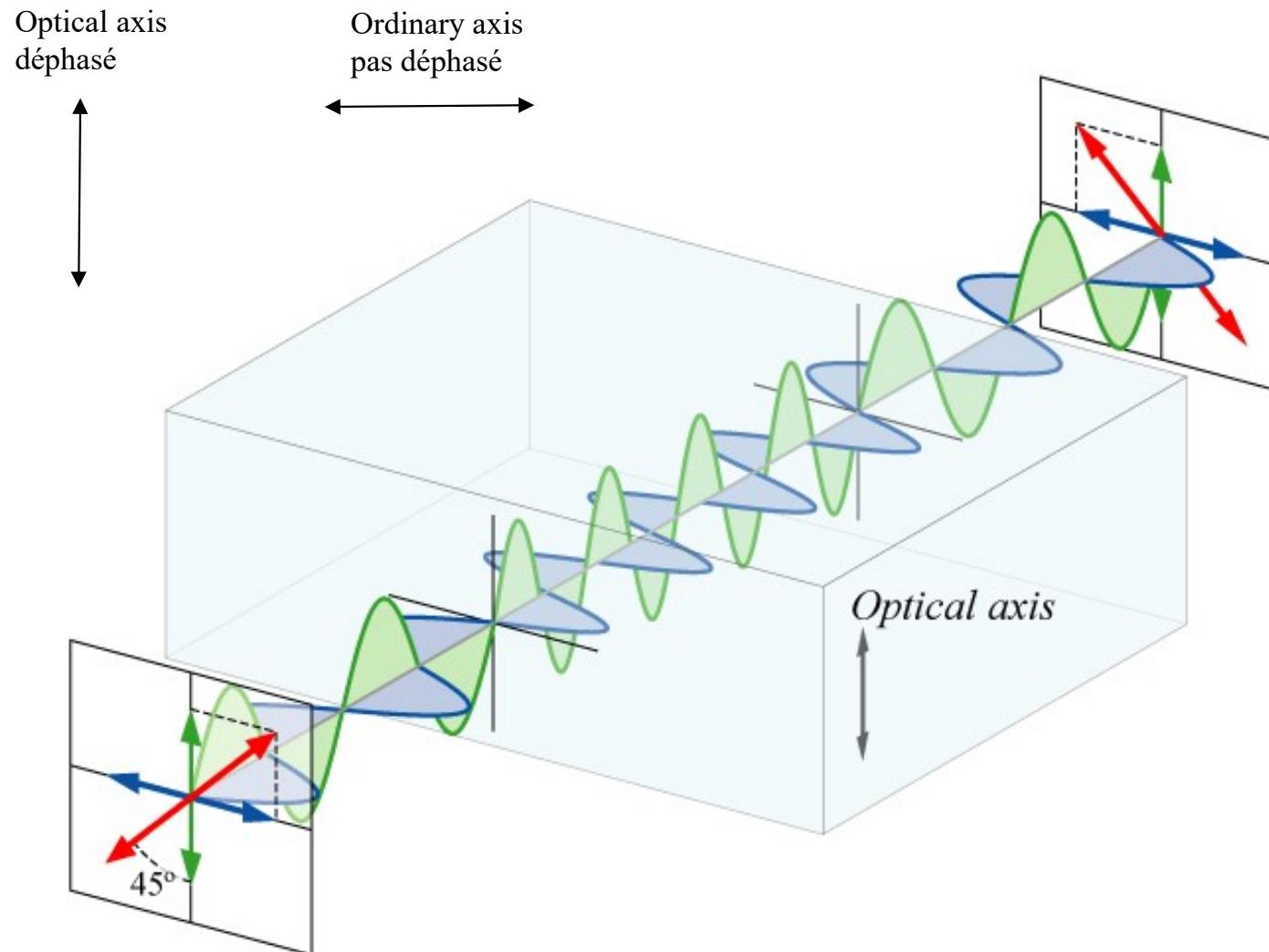
Représentation d'un polariseur sur la sphère de Bloch



$$\sigma_\alpha = n_X \cdot \sigma_X + n_Z \cdot \sigma_Z = \vec{n} \cdot \vec{\sigma}$$

Single Qubit: Rotation de la polarisation

Lame biréfringente



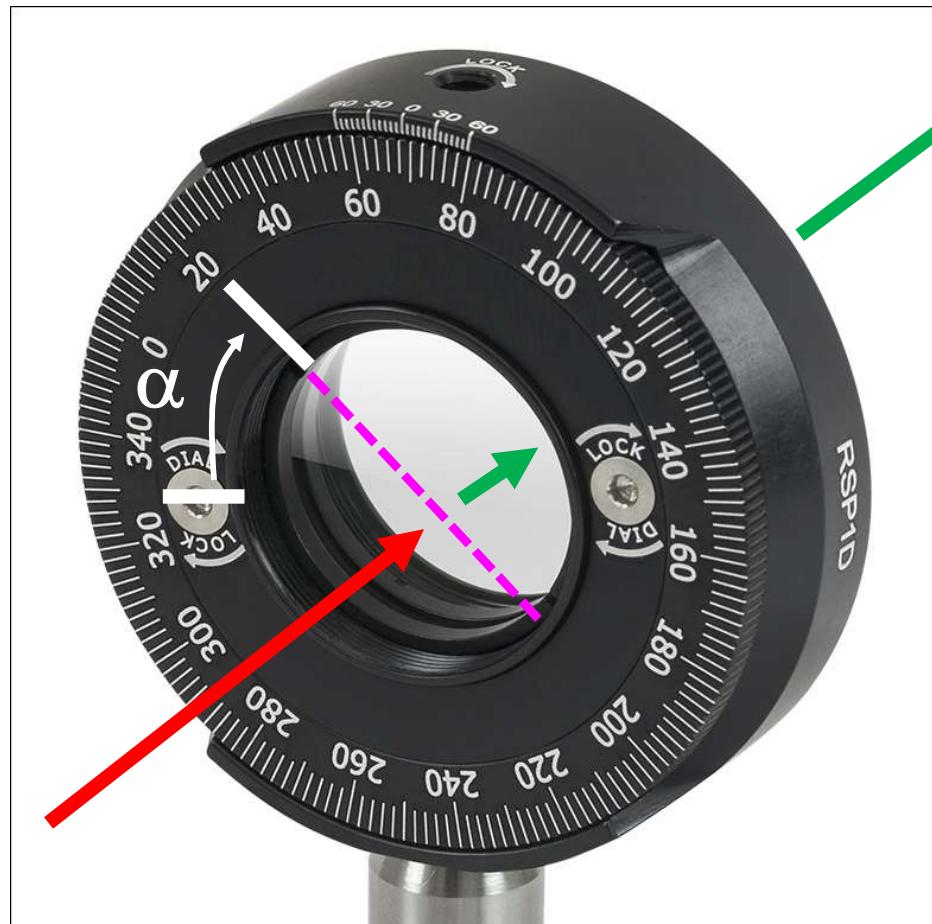
Différence de phase entre les deux axes principaux

Paramètres:

- Épaisseur \rightarrow rotation δ
- Orientation \rightarrow axe α

https://fr.wikipedia.org/wiki/Lame_%C3%A0_retard

Lame biréfringente



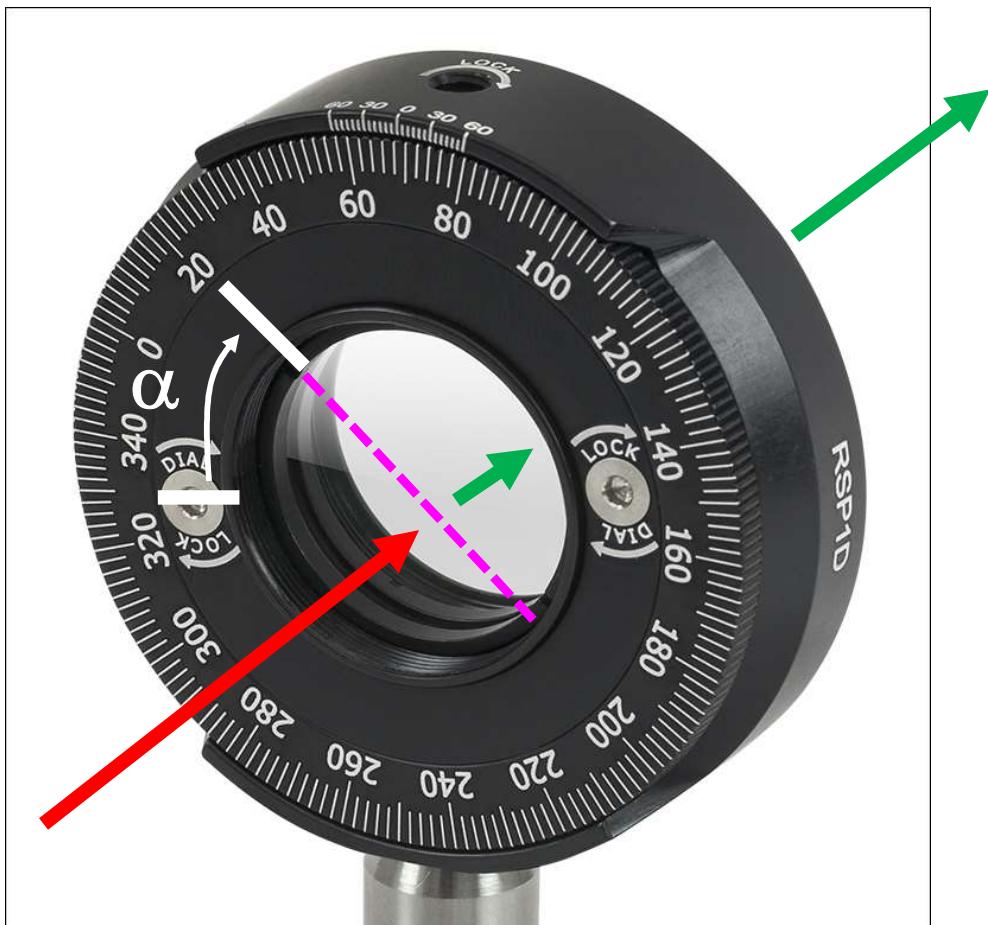
Angle de positionnement = α

Déphasage axe ordinaire/extrordinnaire = δ

(épaisseur $\lambda/2 \rightarrow \delta=\pi$)

(épaisseur $\lambda/4 \rightarrow \delta=\pi/2$)

Lame biréfringente: projecteurs



Projecteurs:

Ordinary axis

$$P_{+1} = |\varphi_{+1}\rangle \cdot \langle \varphi_{+1}| = \begin{pmatrix} \cos^2(\alpha) & \cos(\alpha)\sin(\alpha) \\ \cos(\alpha)\sin(\alpha) & \sin^2(\alpha) \end{pmatrix}$$

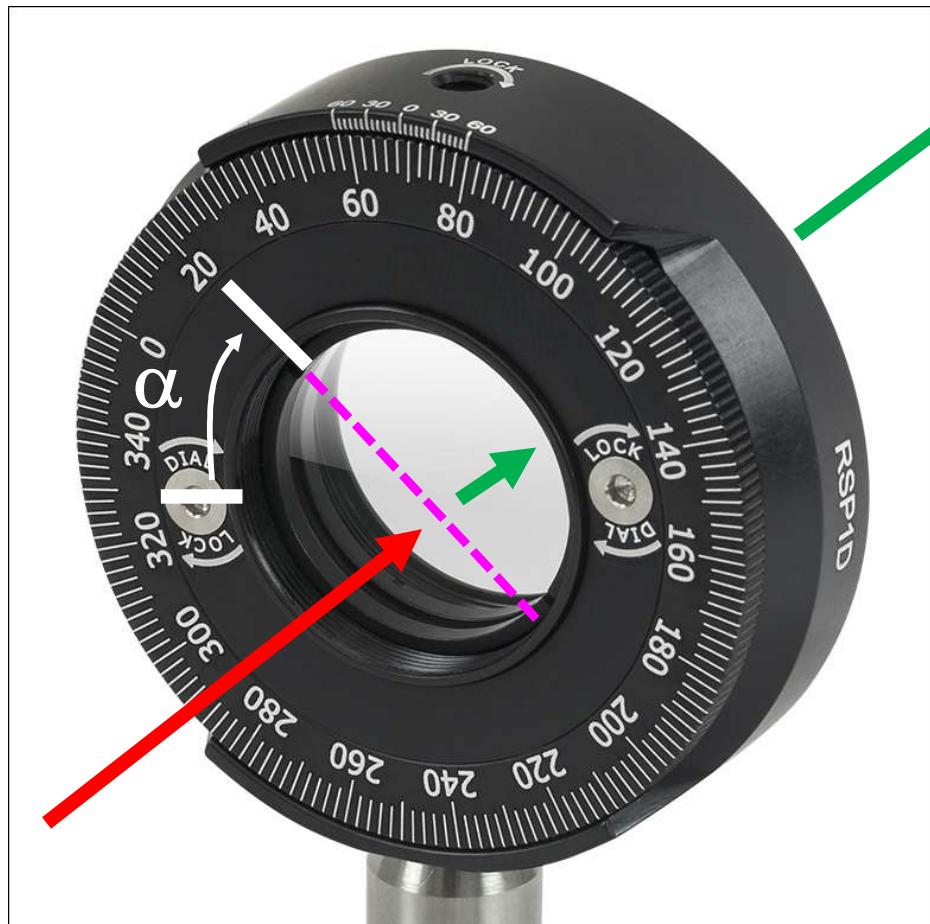
Sans de shift de phase

Optical axis

$$P_{-1} = |\varphi_{-1}\rangle \cdot \langle \varphi_{-1}| = \begin{pmatrix} \sin^2(\alpha) & -\cos(\alpha)\sin(\alpha) \\ -\cos(\alpha)\sin(\alpha) & \cos^2(\alpha) \end{pmatrix}$$

Phase shift: $e^{i\delta}$

Lame biréfringente : opérateur



Rotation optique

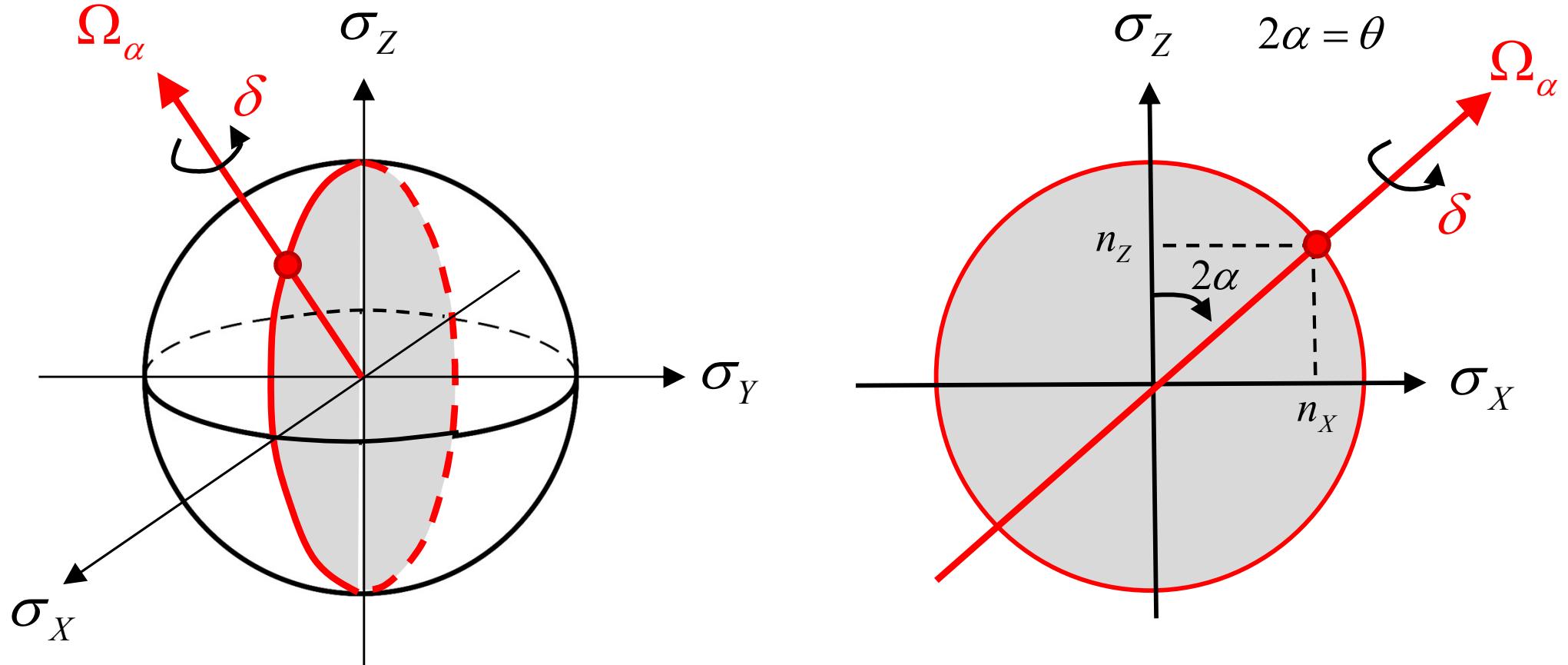
$$\Omega_\alpha = +1 \cdot P_{+1} + e^{i\delta} \cdot P_{-1}$$

...

...

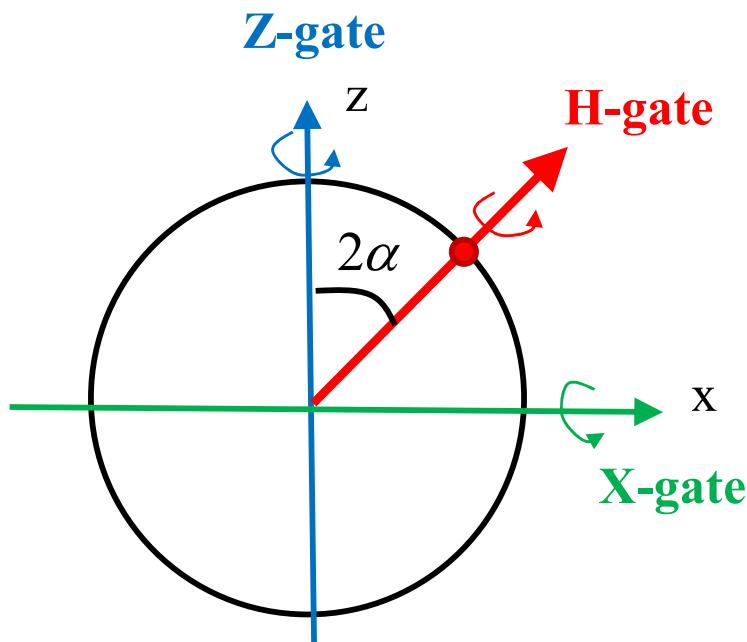
$$\Omega_\alpha = e^{i\frac{\delta}{2}} \cdot \left(\cos\left(\frac{\delta}{2}\right) - i \sin\left(\frac{\delta}{2}\right) \cdot (n_x \cdot \sigma_x + n_z \cdot \sigma_z) \right)$$

Représentation d'une lame biréfringente sur la sphère de Bloch



$$\Omega_\alpha = e^{i\frac{\delta}{2}} \cdot \left(\cos\left(\frac{\delta}{2}\right) - i \sin\left(\frac{\delta}{2}\right) \cdot (n_x \cdot \sigma_x + n_z \cdot \sigma_z) \right)$$

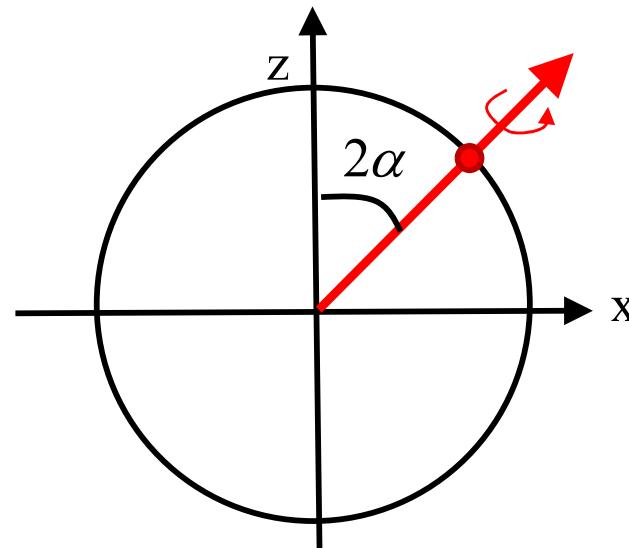
Half Wave Plate (HWP)



Rotation de 180^0

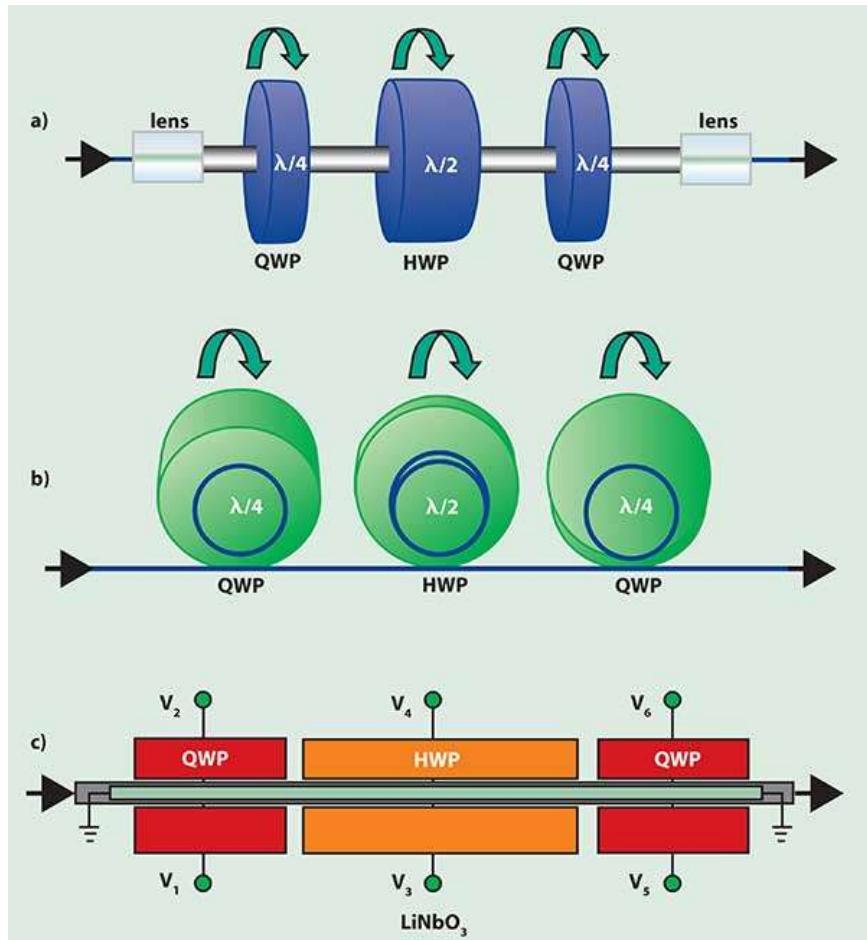
$$\Omega_\alpha = \begin{pmatrix} \cos(2\alpha) & \sin(2\alpha) \\ \sin(2\alpha) & -\cos(2\alpha) \end{pmatrix}$$

Quater Wave Plate (QWP)

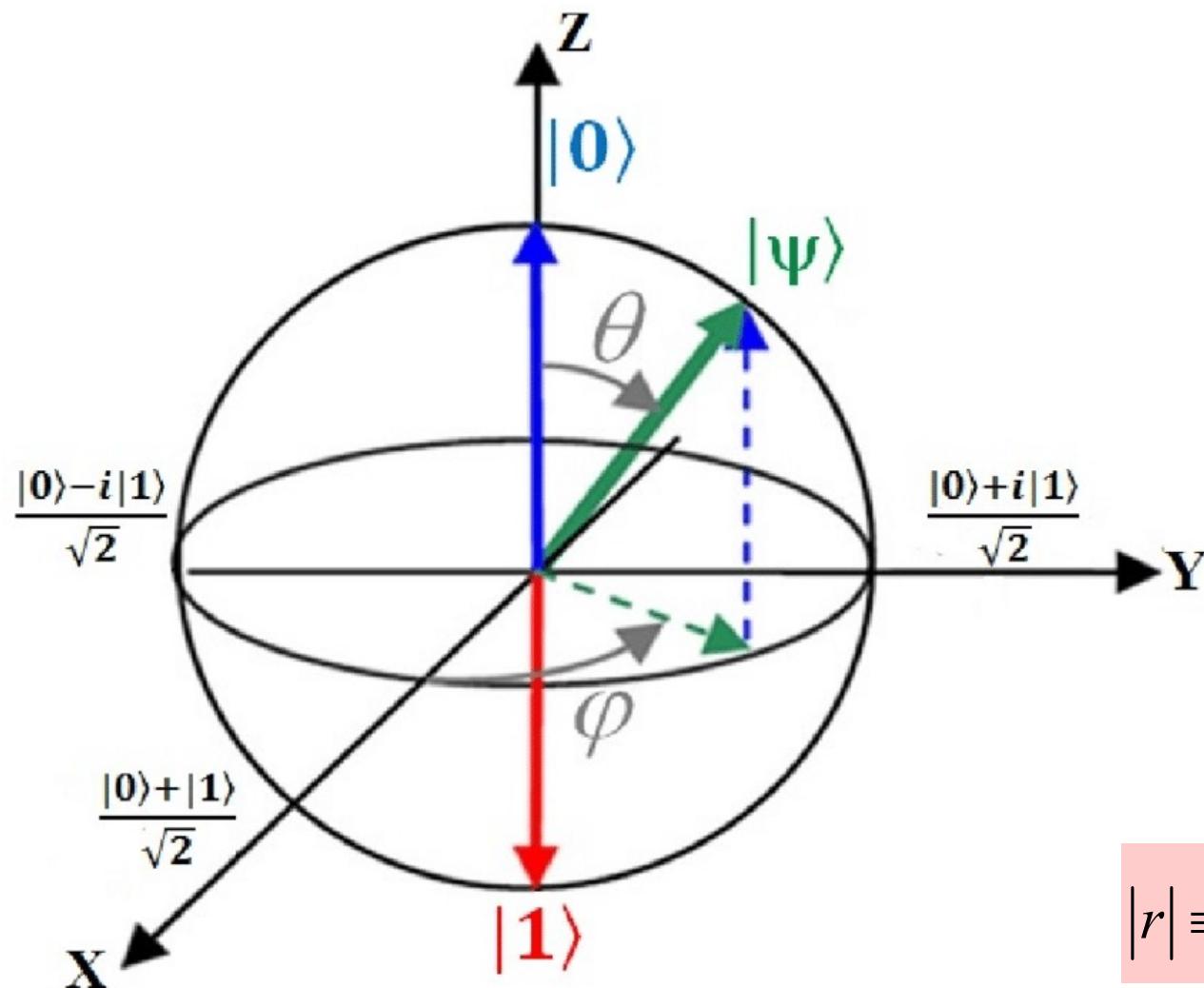


Rotation de 90^0

$$\Omega_\alpha = \begin{pmatrix} \cos^2(\alpha) + i \sin^2(\alpha) & \cos(\alpha) \sin(\alpha) \cdot (1-i) \\ \cos(\alpha) \sin(\alpha) \cdot (1-i) & \sin^2(\alpha) + i \cos^2(\alpha) \end{pmatrix}$$



Degré de polarisation de la lumière

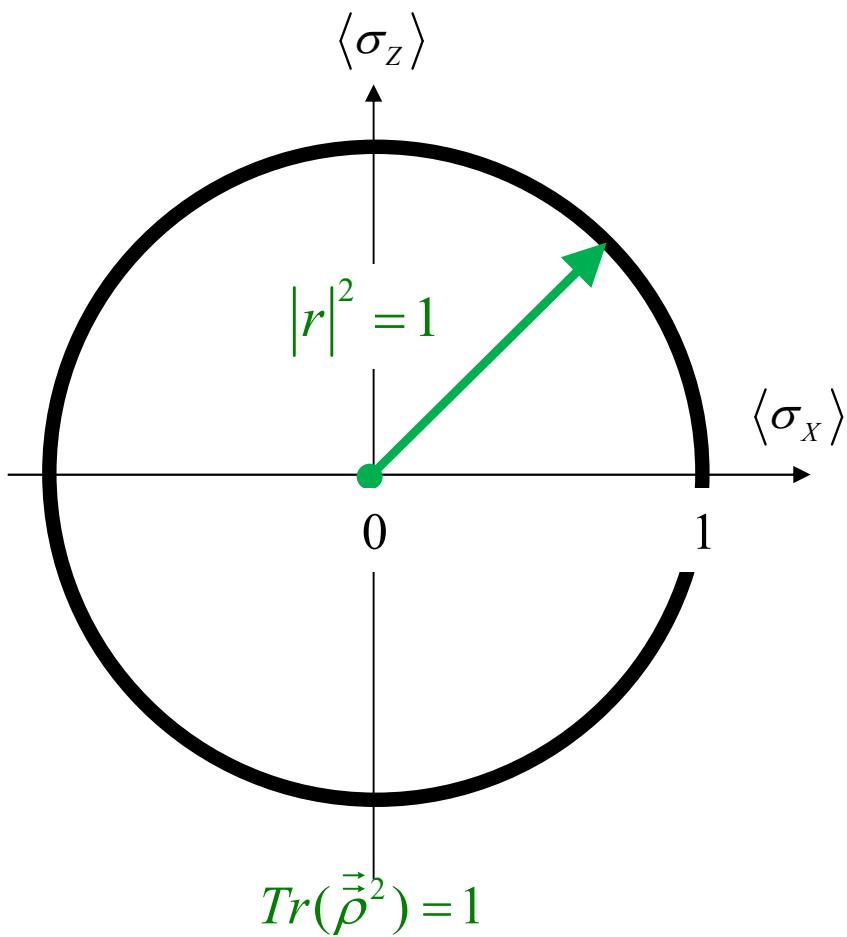


$$|\psi\rangle = \begin{pmatrix} \langle \sigma_x \rangle \\ \langle \sigma_y \rangle \\ \langle \sigma_z \rangle \end{pmatrix} = \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix}$$

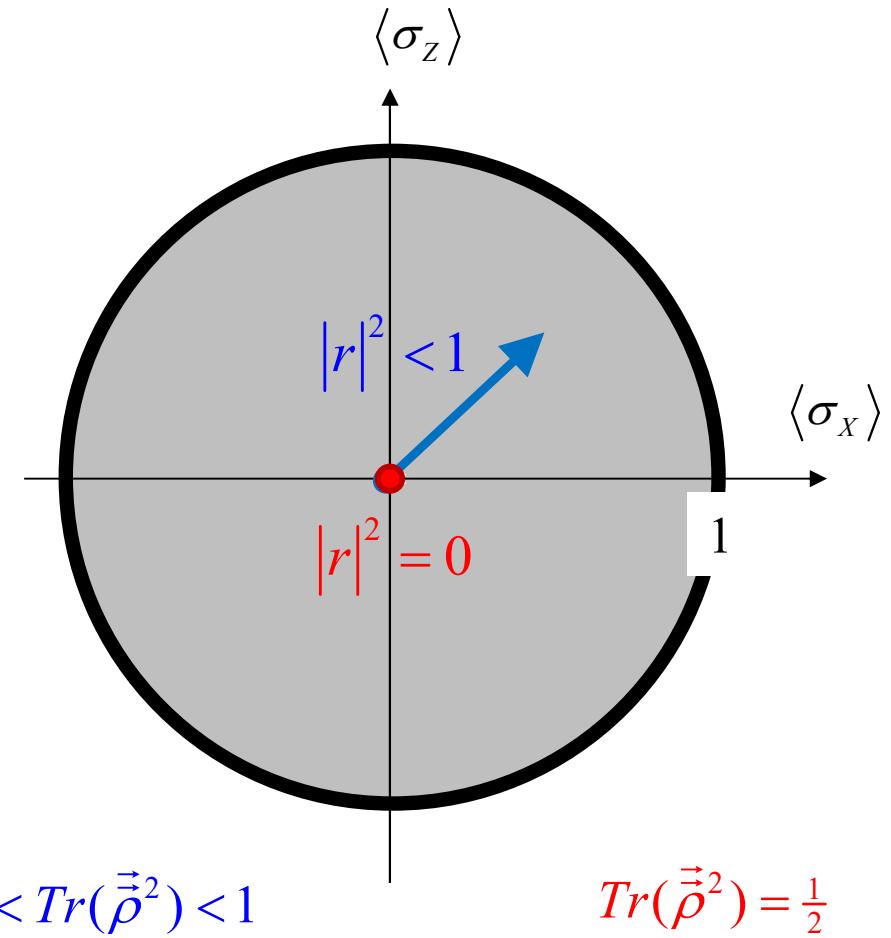
Etat superposé:

$$|r| \equiv \sqrt{\langle \sigma_x \rangle^2 + \langle \sigma_y \rangle^2 + \langle \sigma_z \rangle^2} = 1$$

Totalement polarisé



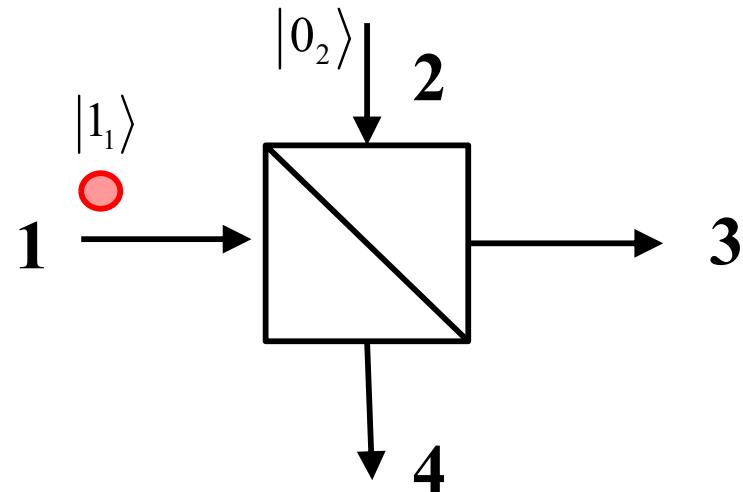
Partiellement polarisé
Non-polarisé



Beam Splitter

et

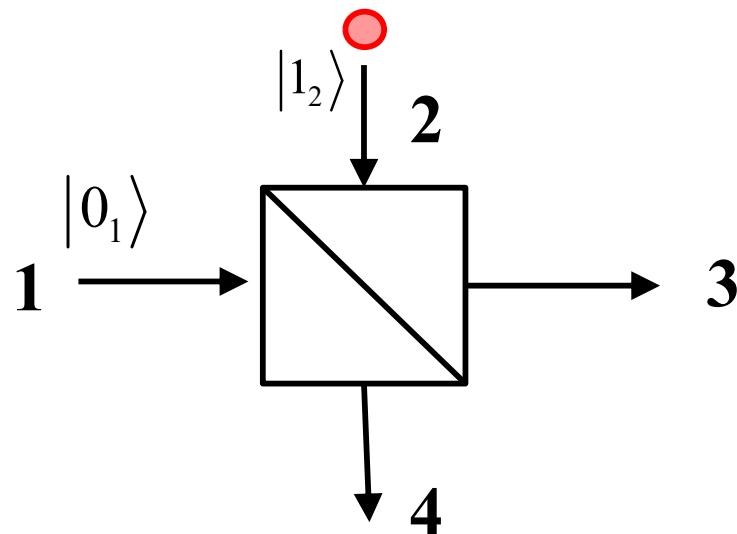
Comptage de photons



Un photon à l'entrée «1»

$$|IN\rangle = |1_1, 0_2\rangle = a_1^+ \cdot |0_1, 0_2\rangle$$

$$\Rightarrow |OUT\rangle = (t \cdot a_3^+ + r \cdot a_4^+) \cdot |0_3, 0_4\rangle$$



Un photon à l'entrée «1»

$$|IN\rangle = |1_1, 0_2\rangle = a_1^+ \cdot |0_1, 0_2\rangle$$

$$\Rightarrow |OUT\rangle = (t \cdot a_3^+ + r \cdot a_4^+) \cdot |0_3, 0_4\rangle$$

Un photon à l'entrée «2»

$$|IN\rangle = |0_1, 1_2\rangle = a_2^+ \cdot |0_1, 0_2\rangle$$

$$\Rightarrow |OUT\rangle = (-\bar{r} \cdot a_3^+ + t \cdot a_4^+) \cdot |0_3, 0_4\rangle$$



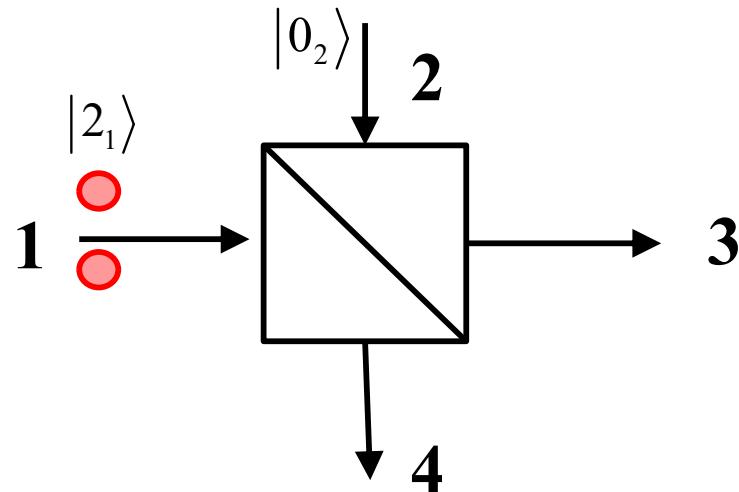
Conservation énergie



$$a_1^+ \rightarrow t \cdot a_3^+ + r \cdot a_4^+$$

$$a_2^+ \rightarrow -\bar{r} \cdot a_3^+ + t \cdot a_4^+$$





2 photons à l'entrée «1»

$$|IN\rangle = |2_1, 0_2\rangle = \frac{(a_1^+)^2}{\sqrt{2}} \cdot |0_1, 0_2\rangle$$

$$|OUT\rangle = \frac{(t \cdot a_3^+ + r \cdot a_4^+)^2}{\sqrt{2}} \cdot |0_3, 0_4\rangle$$

$$= t^2 \cdot |2_3, 0_4\rangle + \sqrt{2} \cdot r \cdot t \cdot |1_3, 1_4\rangle + r^2 \cdot |0_3, 2_4\rangle$$

Distribution en sortie

$$P_{2,0} = |t|^4$$

$$P_{1,1} = 2|t|^2|r|^2 \quad \Rightarrow \quad$$

$$P_{0,2} = |r|^4$$

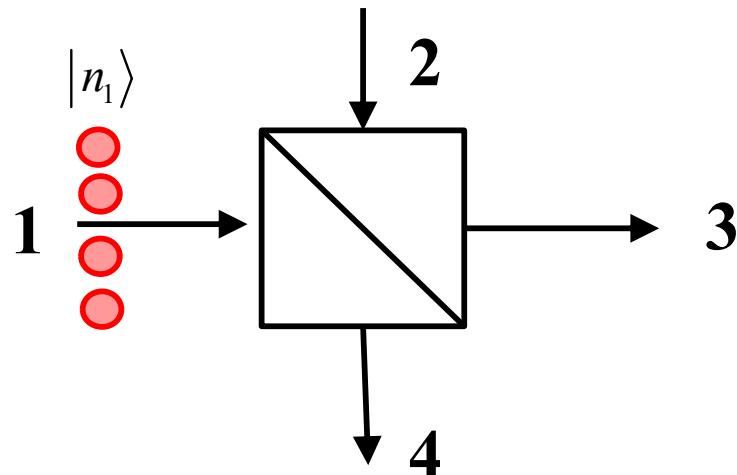
Moyennes

$$\begin{aligned} N3 &= 2 \cdot P_{2,0} + 1 \cdot P_{1,1} = 2 \cdot |t|^2 \\ N4 &= 1 \cdot P_{1,1} + 2 \cdot P_{0,2} = 2 \cdot |r|^2 \end{aligned}$$

Cas spécifique $|t|^2 = |r|^2 = \frac{1}{2}$

$$P_{2,0} = P_{0,2} = \frac{1}{4} \quad N3 = 1 \quad N4 = 1$$

$$P_{1,1} = \frac{1}{2}$$



n photons à l'entrée «1»

$$|IN\rangle = |n_1, 0_2\rangle = \frac{(a_1^+)^n}{\sqrt{n!}} \cdot |0_1, 0_2\rangle$$

$$\Rightarrow |OUT\rangle = \frac{(t \cdot a_3^+ + r \cdot a_4^+)^n}{\sqrt{n!}} \cdot |0_3, 0_4\rangle$$

Binôme de Newton →

$$= \sum_{k=0}^n \sqrt{\frac{n!}{k!(n-k)!}} \cdot r^k \cdot t^{(n-k)} \cdot |(n-k)_3, k_4\rangle$$

Distribution en sortie

$$P_{(n-k),k} = \frac{n!}{k!(n-k)!} \cdot |r|^{2k} \cdot |t|^{2(n-k)}$$

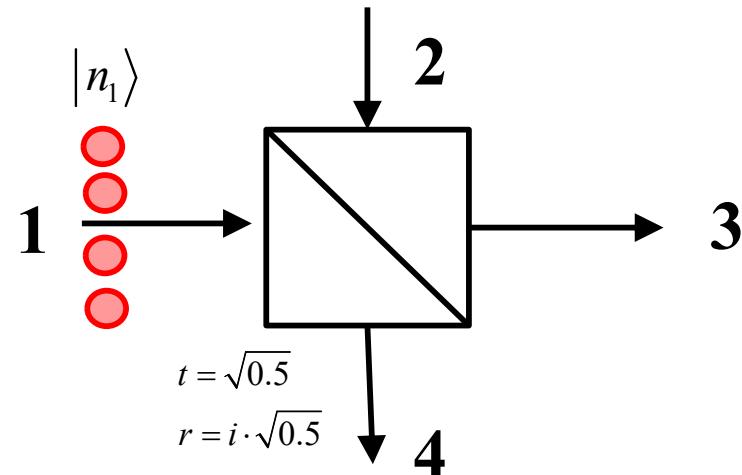
(distribution binomiale)

Moyennes

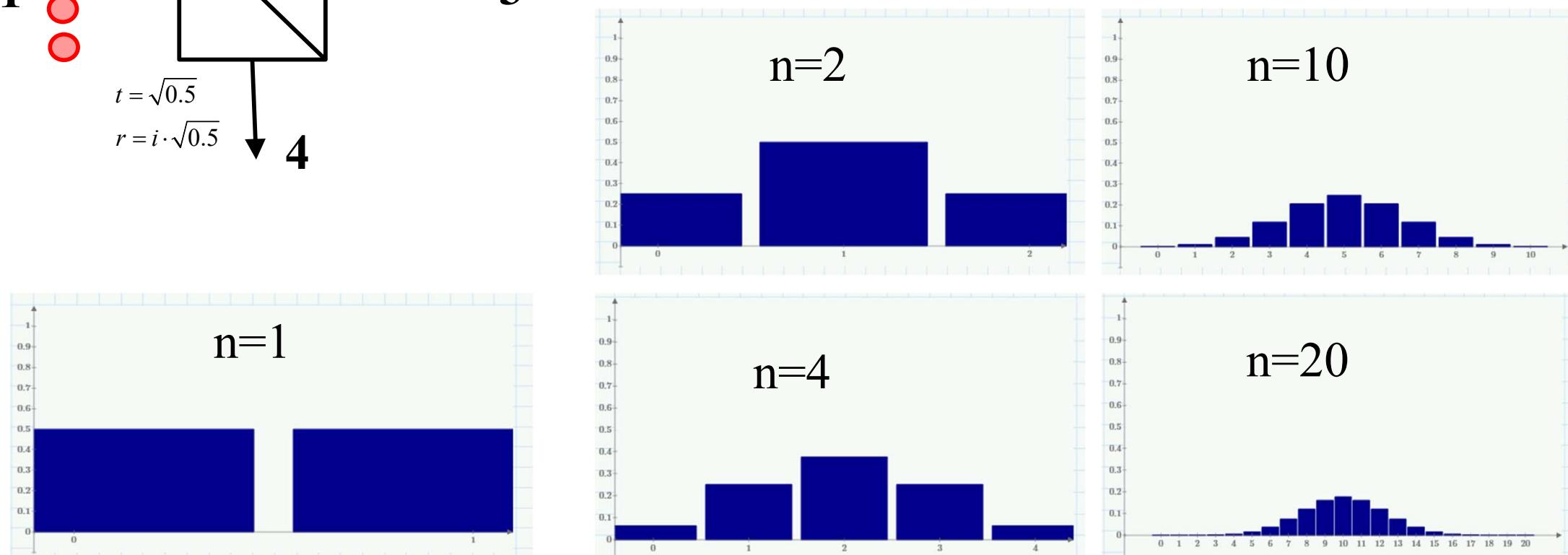
$$N3 = \sum_{k=0}^n (n-k) \cdot P_{(n-k),k} = n \cdot |t|^2$$

$$N4 = \sum_{k=0}^n k \cdot P_{(n-k),k} = n \cdot |r|^2$$

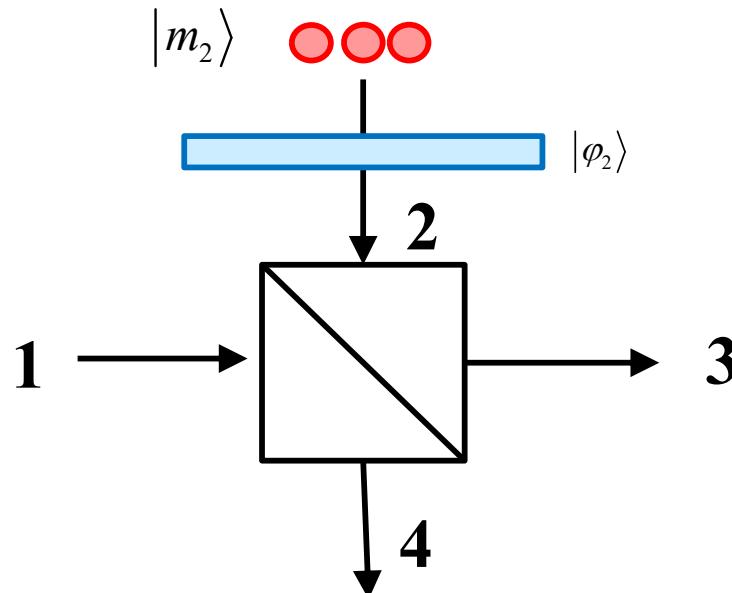
Exemples: n-photons à l'entrée 1



Sorties 3 ou 4



distribution binomiale



m photons à l'entrée «2»

$$|IN\rangle = |0_1, m_2\rangle = \frac{(a_2^+)^m}{\sqrt{m!}} \cdot |0_1, 0_2\rangle$$

$$\Rightarrow |OUT\rangle = (e^{i\varphi_2})^m \cdot \frac{(-\bar{r} \cdot a_3^+ + t \cdot a_4^+)^m}{\sqrt{m!}} \cdot |0_3, 0_4\rangle$$

Binôme de Newton \rightarrow

$$= e^{im\varphi_2} \cdot \sum_{k=0}^m (-1)^{(m-k)} \sqrt{\frac{m!}{k!(m-k)!}} \cdot t^k \cdot \bar{r}^{(m-k)} \cdot |(m-k)_3, k_4\rangle$$

Distribution en sortie

$$P_{(m-k),k} = \frac{m!}{k!(m-k)!} \cdot |t|^{2k} \cdot |\bar{r}|^{2(m-k)}$$

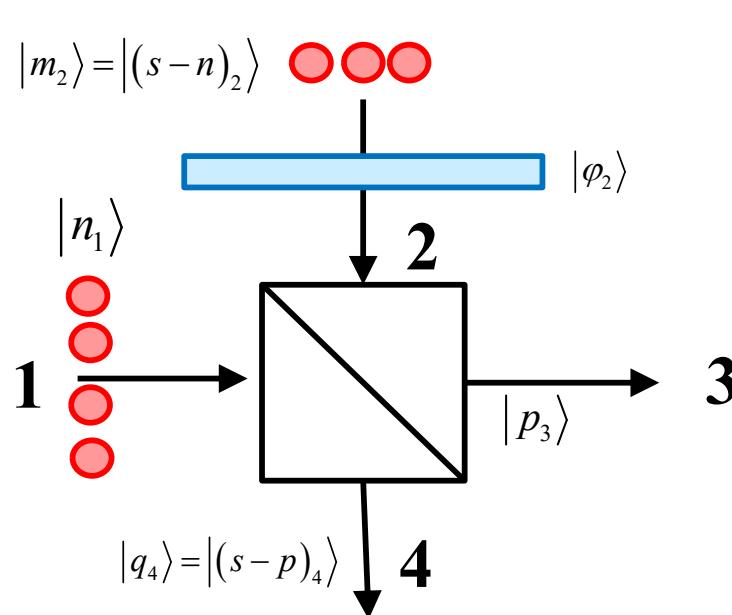
(distribution binomiale)

Moyennes

$$N3 = \sum_{k=0}^m (m-k) \cdot P_{(m-k),k} = m \cdot |\bar{r}|^2$$

$$N4 = \sum_{k=0}^m k \cdot P_{(m-k),k} = m \cdot |t|^2$$

s photons en total dont
n photons à l'entrée «1»



$$\begin{aligned}
 |IN\rangle &= \left| n_1, \binom{s-n}{2} \right\rangle = \frac{\left(a_1^+ \right)^n}{\sqrt{n!}} \cdot \frac{\left(a_2^+ \right)^{s-n}}{\sqrt{(s-n)!}} \cdot \left| 0_1, 0_2 \right\rangle \\
 |OUT\rangle &= \frac{\left(t \cdot a_3^+ + r \cdot a_4^+ \right)^n}{\sqrt{n!}} \cdot \left(e^{i\varphi_2} \right)^{s-n} \cdot \frac{\left(-\bar{r} \cdot a_3^+ + t \cdot a_4^+ \right)^{s-n}}{\sqrt{(s-n)!}} \cdot \left| 0_3, 0_4 \right\rangle \\
 &= e^{i(s-n)\cdot\varphi} \cdot \sum_{p=0}^{s-n} \alpha_{n,p} \cdot \left| p_3, \binom{s-p}{4} \right\rangle
 \end{aligned}$$

$$\alpha_{n,p} = \sqrt{n!} \cdot \sqrt{(s-n)!} \cdot \sqrt{p!} \cdot \sqrt{(s-p)!} \cdot \sum_{k=\max(0,(n-p))}^{\min(n,(s-p))} \left(\frac{1}{k! \cdot (n-k)! \cdot (s-p-k)! \cdot (p+k-n)!} \cdot (-1)^{p+k-n} \cdot t^{n+s-p-2k} \cdot r^k \cdot \bar{r}^{p+k-n} \right)$$

Distribution en sortie

$$P_{p,(s-p)} = |\alpha_{n,p}|^2$$



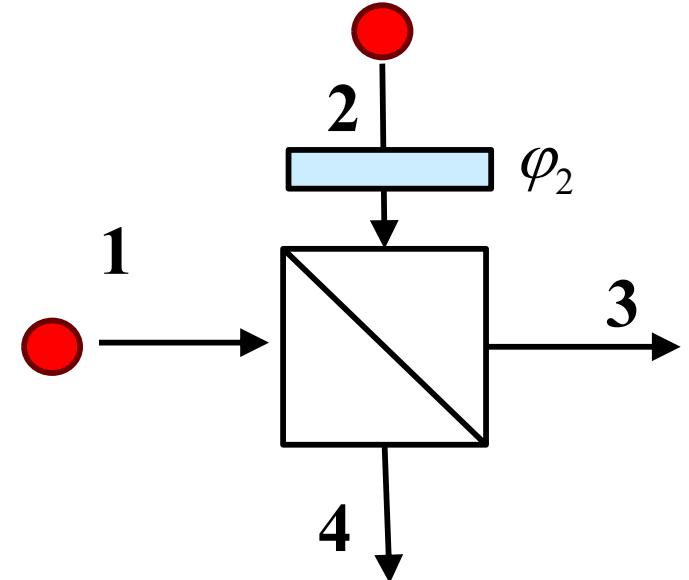
Moyennes

$$N3 = \sum_{p=0}^s p \cdot P_{p,(s-p)}$$

$$N4 = \sum_{p=0}^s (s-p) \cdot P_{p,(s-p)}$$

$$|IN\rangle = |1_1, 1_2\rangle = (a_1^+)(a_2^+) \cdot |0_1, 0_2\rangle$$

$$\begin{aligned} |OUT\rangle &= (t \cdot a_3^+ + r \cdot a_4^+) \cdot (-\bar{r} \cdot a_3^+ + t \cdot a_4^+) \cdot e^{i\varphi_2} \cdot |0_3, 0_4\rangle \\ &= \left(t \cdot \bar{r} \cdot (a_3^+)^2 + (t^2 - |r|^2) \cdot a_3^+ a_4^+ + t \cdot r \cdot (a_4^+)^2 \right) \cdot e^{i\varphi_2} \cdot |0_3, 0_4\rangle \\ &= \left(t \cdot \bar{r} \cdot \sqrt{2} \cdot \frac{(a_3^+)^2}{\sqrt{2}} + (|t|^2 - |r|^2) \cdot a_3^+ a_4^+ + t \cdot r \cdot \sqrt{2} \cdot \frac{(a_4^+)^2}{\sqrt{2}} \right) \cdot e^{i\varphi_2} \cdot |0_3, 0_4\rangle \\ &= e^{i\varphi_2} \cdot \left\{ [t \cdot \bar{r} \cdot \sqrt{2}] \cdot |2_3, 0_4\rangle + [(|t|^2 - |r|^2)] \cdot |1_3, 1_4\rangle + [t \cdot r \cdot \sqrt{2}] \cdot |0_3, 2_4\rangle \right\} \end{aligned}$$



Distribution en sortie

$$P_{2,0} = 2 \cdot |t|^2 \cdot |r|^2$$

$$P_{1,1} = (|t|^2 - |r|^2)^2$$

$$P_{0,2} = 2 \cdot |t|^2 \cdot |r|^2$$



Moyennes

$$N3 = 2 \cdot P_{2,0} + 1 \cdot P_{1,1} = (|t|^2 + |r|^2)^2 = 1$$

$$N4 = 1 \cdot P_{1,1} + 2 \cdot P_{0,2} = (|t|^2 + |r|^2)^2 = 1$$

Ne dépend pas de la phase !!

Cas spécifique $|t|^2 = |r|^2 = \frac{1}{2}$

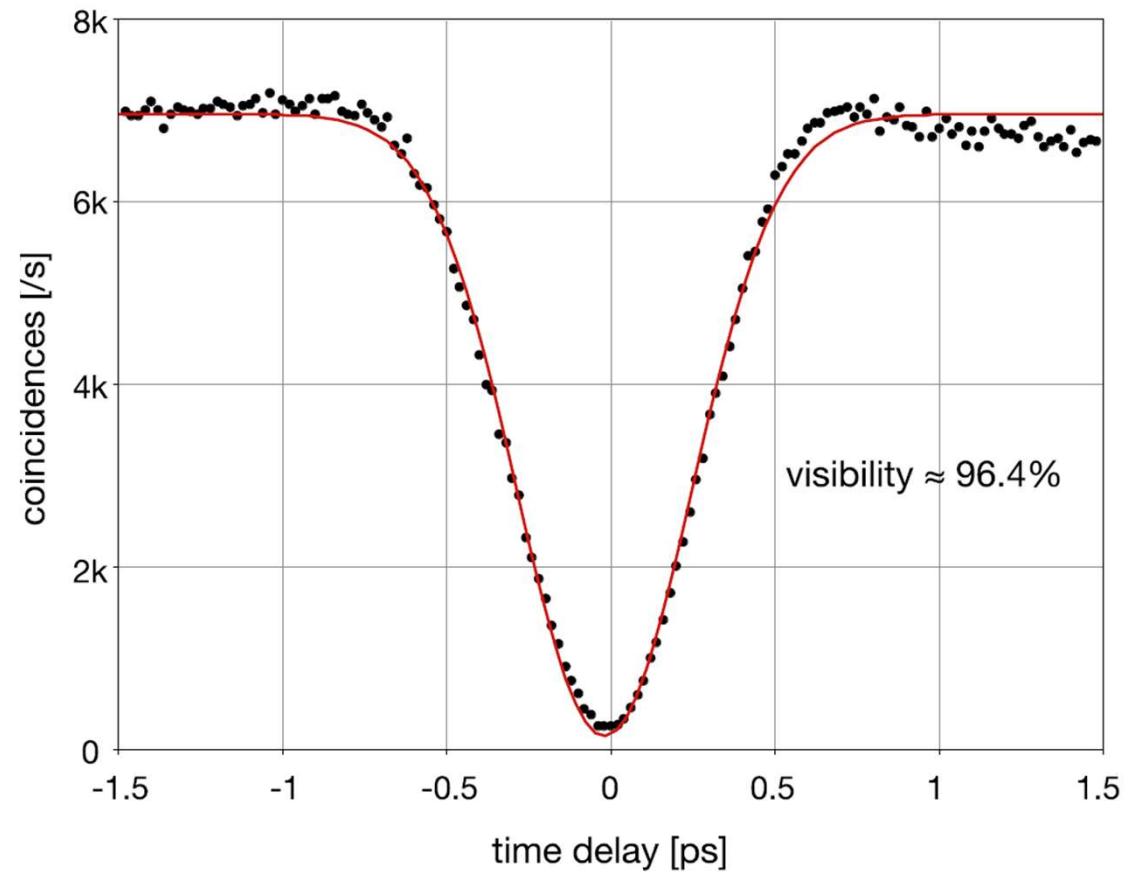
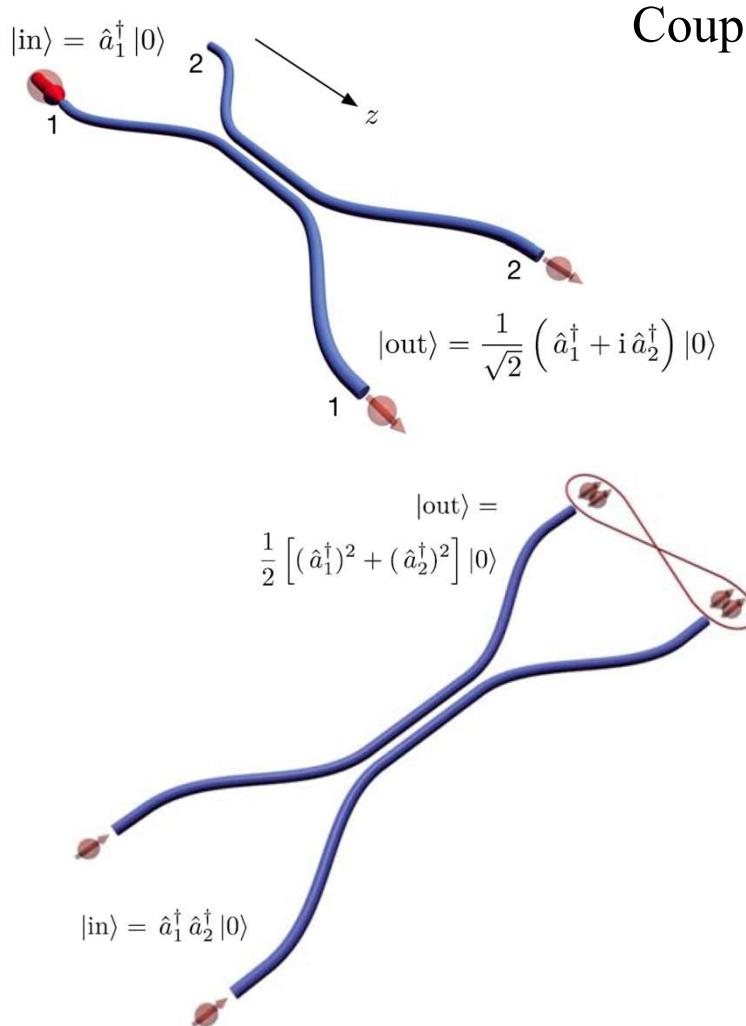
$$P_{2,0} = P_{0,2} = \frac{1}{2}$$

$$P_{1,1} = 0$$



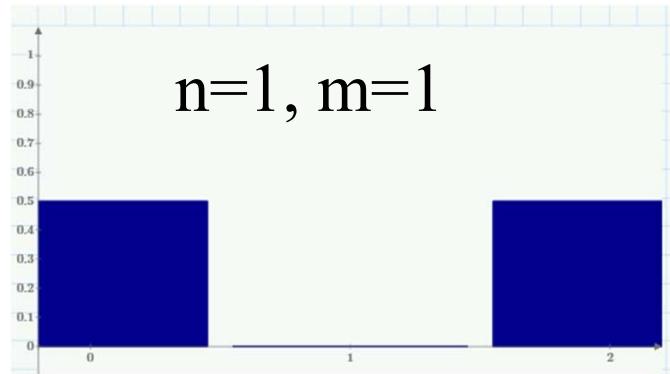
Effet HOM mesuré avec des Coupleurs à fibre monomode

Markus Gräfe and Alexander Szameit 2020 J. Phys. B: At. Mol. Opt. Phys. 53 073001

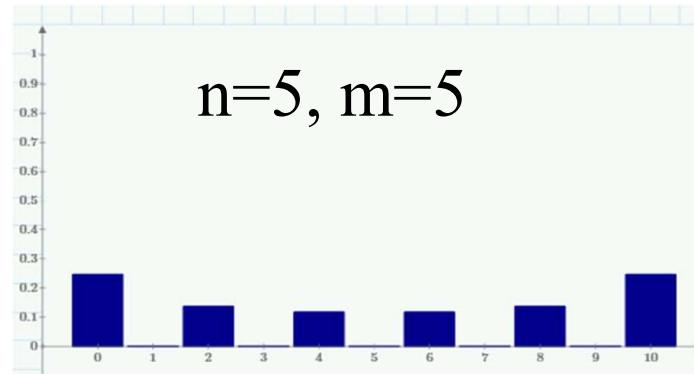


Sorties 3 ou 4

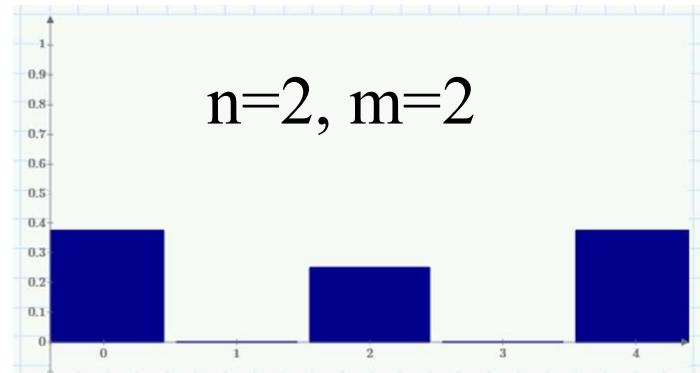
$n=1, m=1$



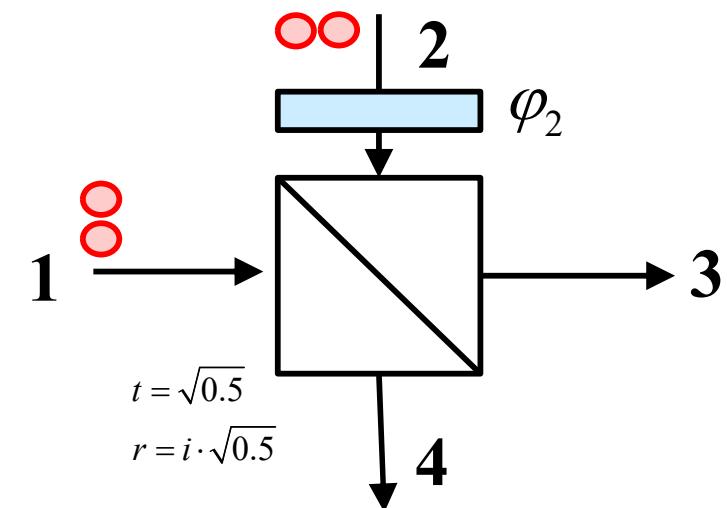
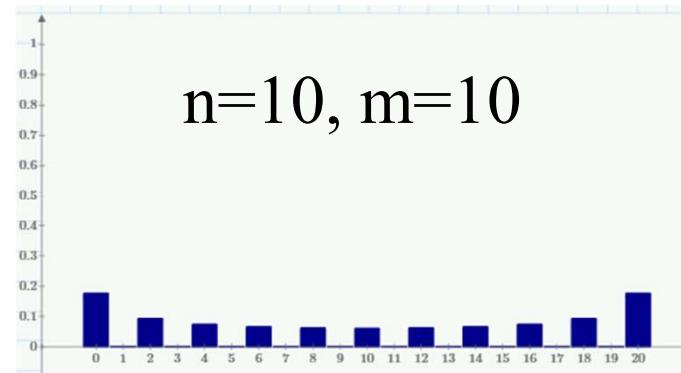
$n=5, m=5$



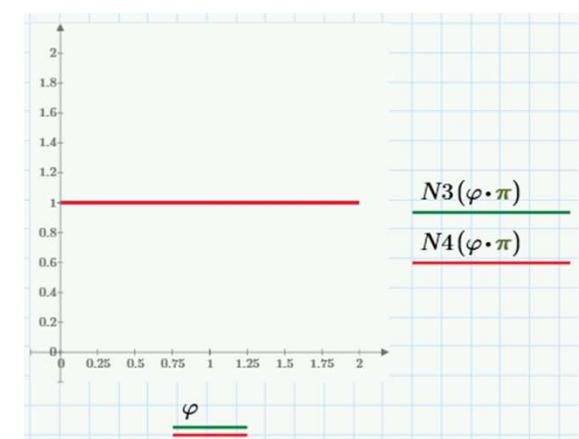
$n=2, m=2$



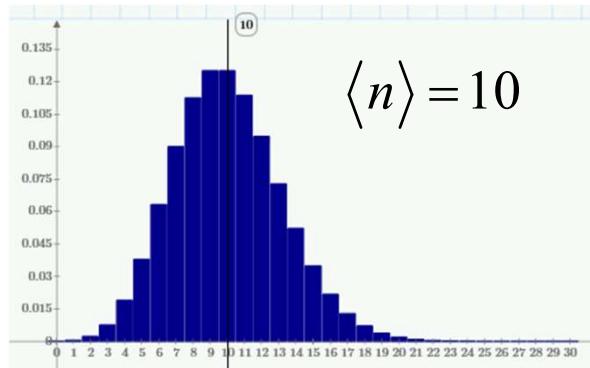
$n=10, m=10$



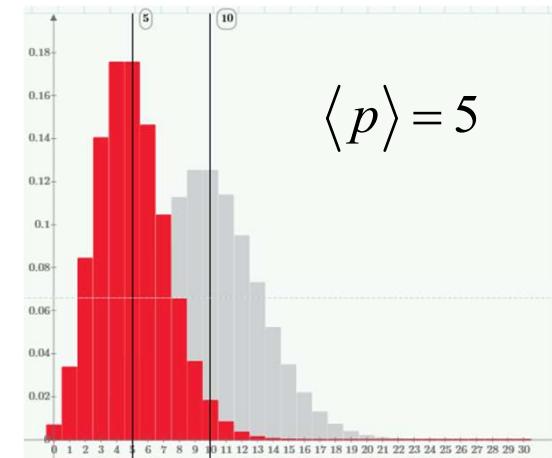
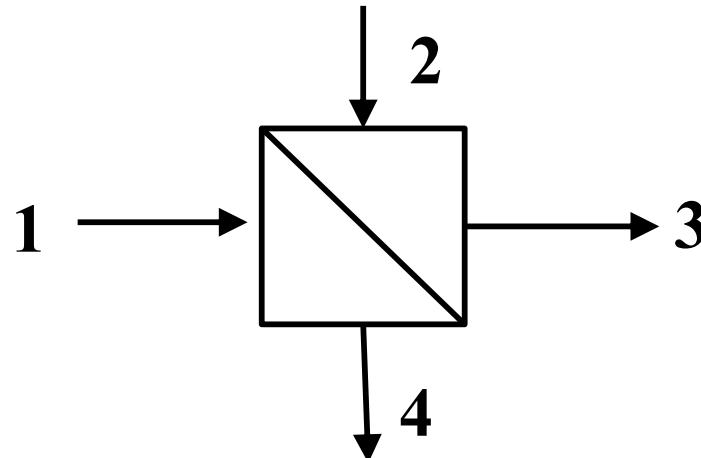
Aucune interférence



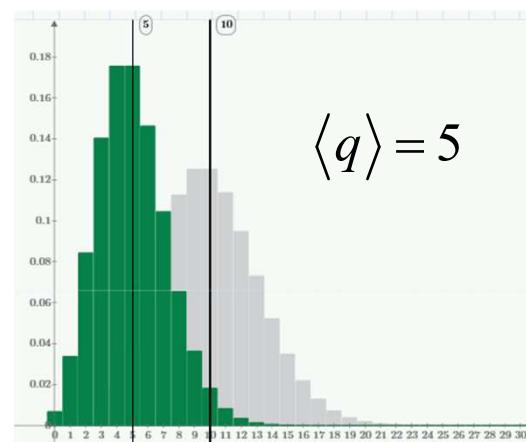
Beam splitter et modes cohérents (1)



Variance $\sigma = \sqrt{\langle n \rangle}$



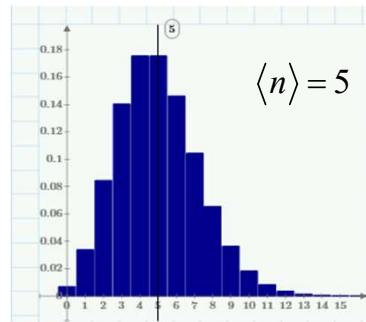
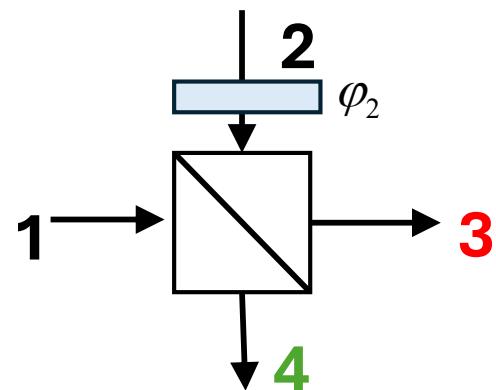
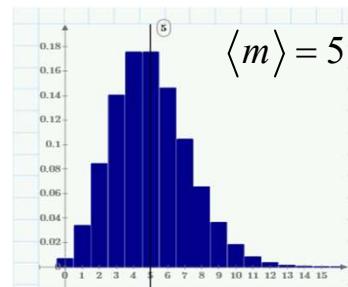
$\langle p \rangle = 5$



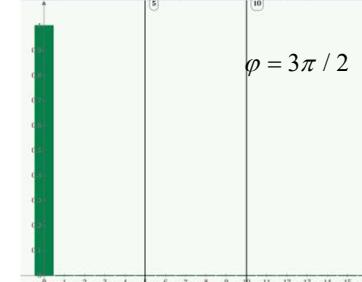
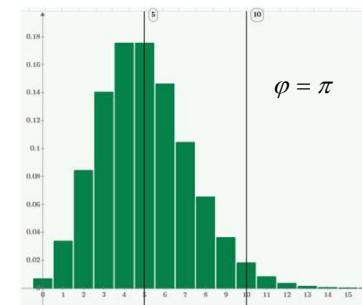
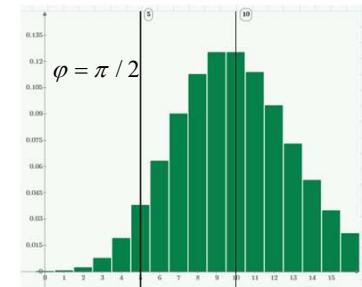
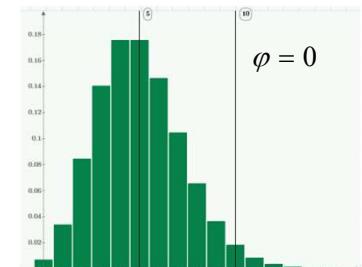
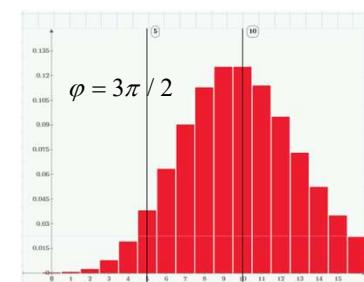
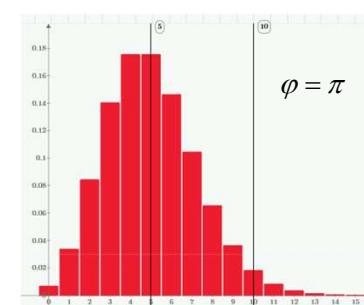
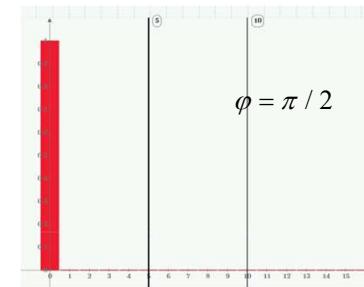
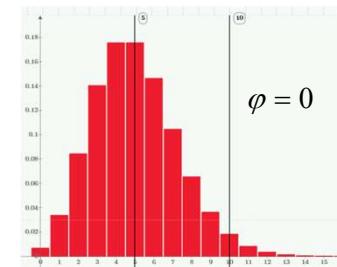
Variance
 $\sigma = \sqrt{\langle q \rangle}$

Les modes cohérents se comportent comme des rayons laser classiques

Les modes cohérents
se comportent comme
des rayons laser classiques

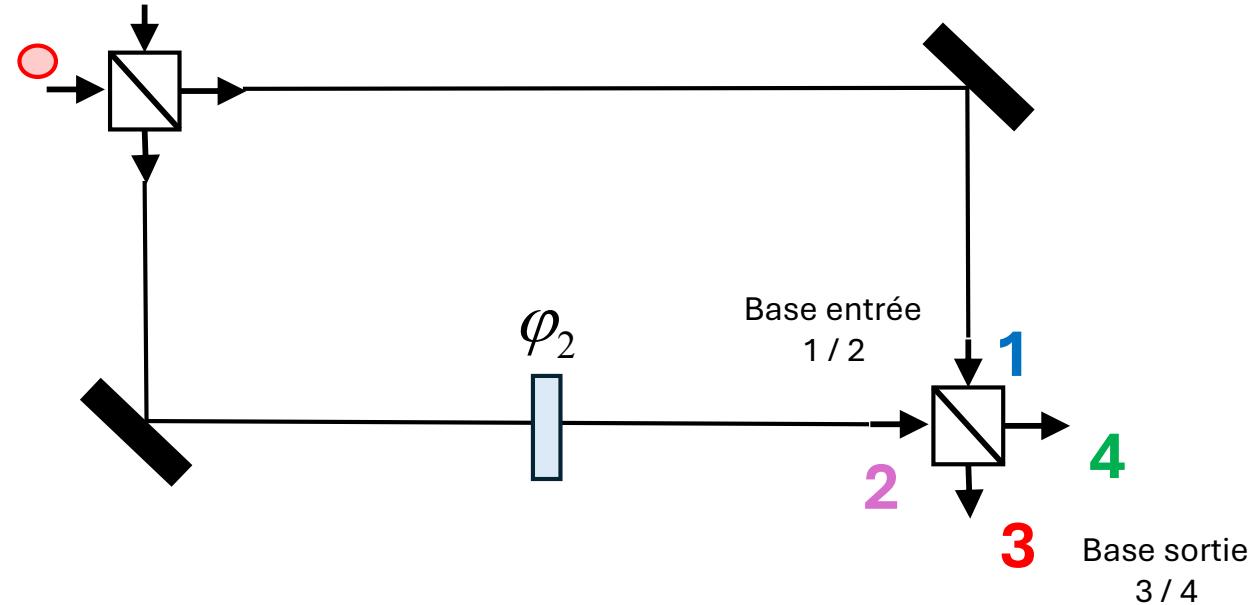


Interférences



Interféromètres et Comptage de photons

Mach Zehnder Interféromètres (MZI): un seul photon à l'entrée

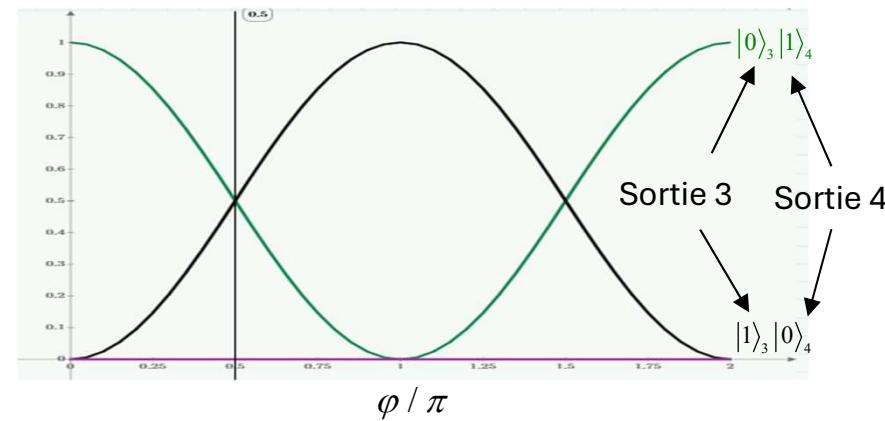
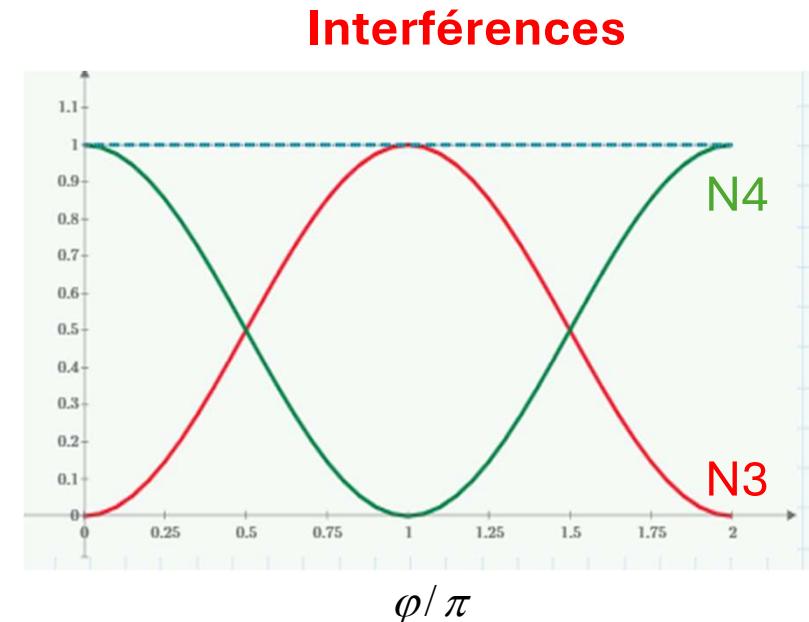


Mode intriqué à l'entrée du second BS

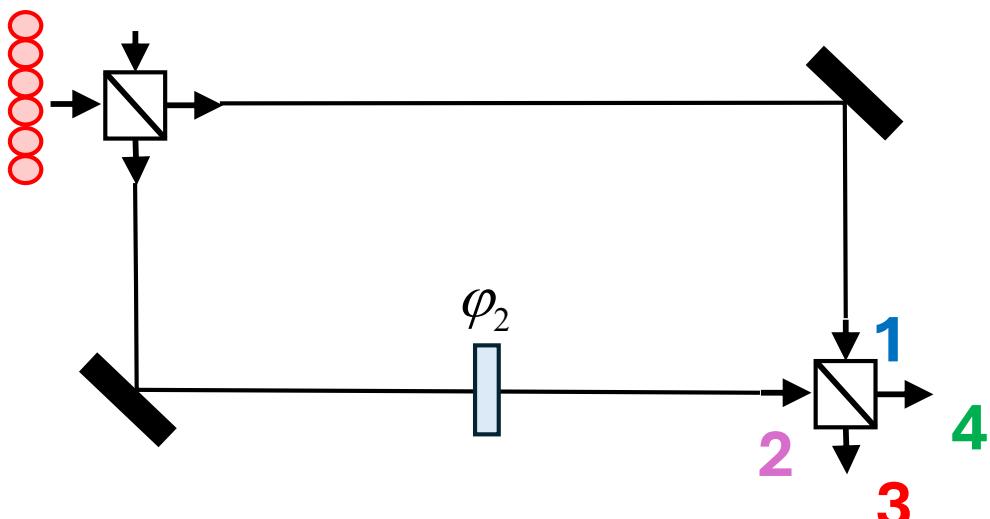
$$\frac{1}{\sqrt{2}} \cdot (|1\rangle_1 |0\rangle_2 + i \cdot |0\rangle_1 |1\rangle_2 e^{i\varphi_2})$$

Entrée 1

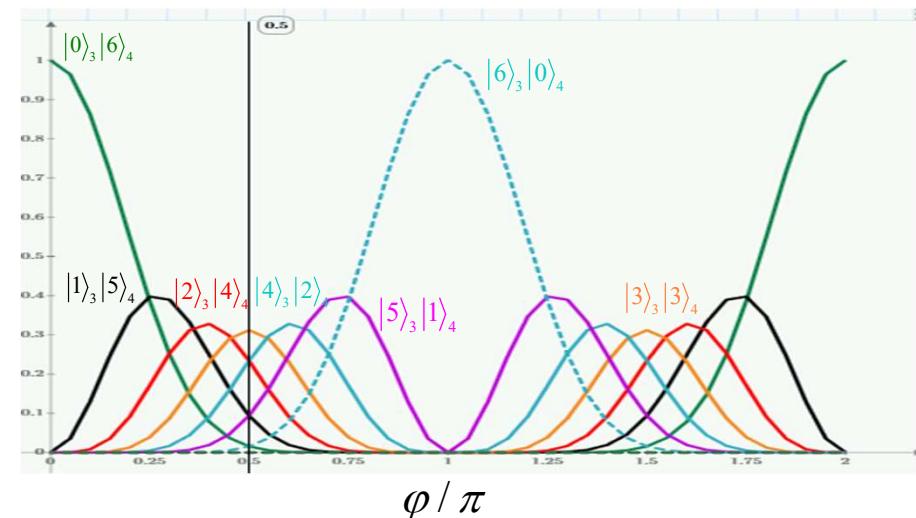
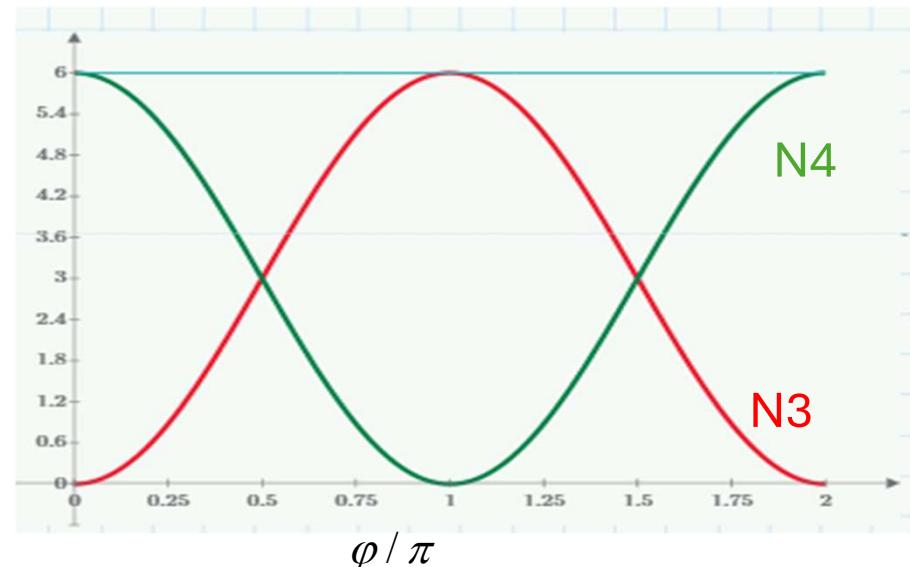
Entrée 2

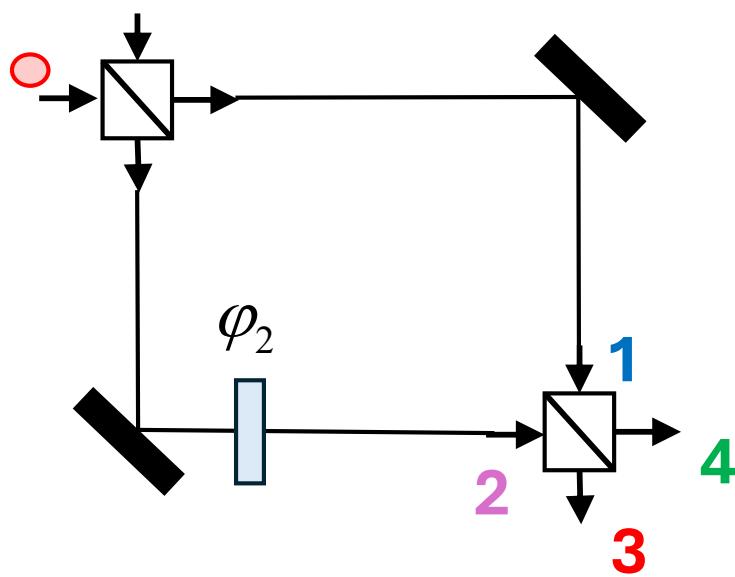


Mach Zehnder Interféromètres (MZI): 6 photons à l'entrée



Interférences

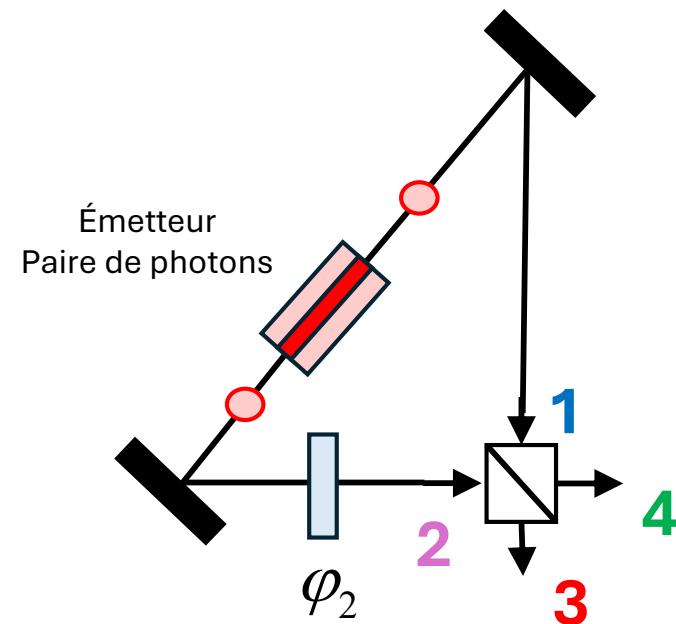




Mode intriqué à l'entrée du second BS

$$\frac{1}{\sqrt{2}} \cdot \left(|1\rangle_1 |0\rangle_2 + |0\rangle_1 |1\rangle_2 e^{i(\varphi_2 + \frac{\pi}{2})} \right)$$

Interférences



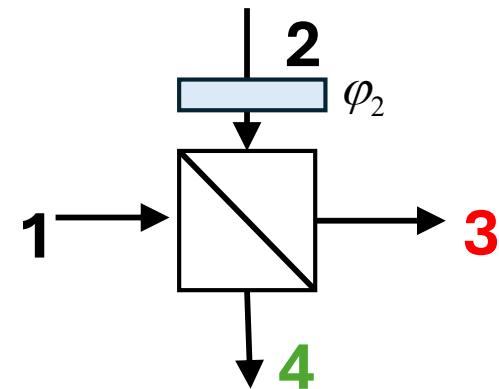
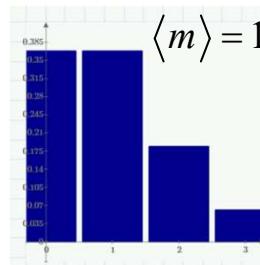
Mode produit à l'entrée du BS

$$|1\rangle_1 |1\rangle_2 e^{i\varphi_2}$$

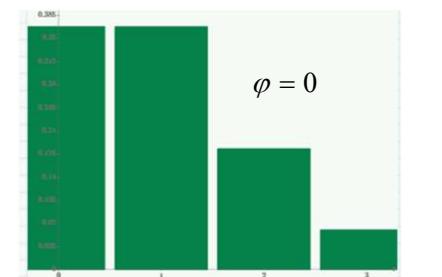
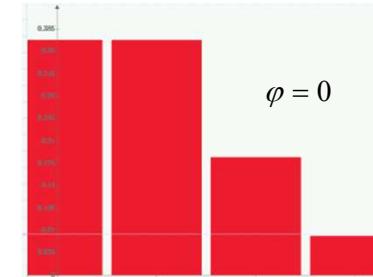
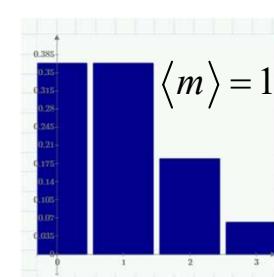
- Aucune interférence
- Effet HOM

Exercice 10-1: Beam splitter et modes cohérents: un mode cohérent à chaque entrée

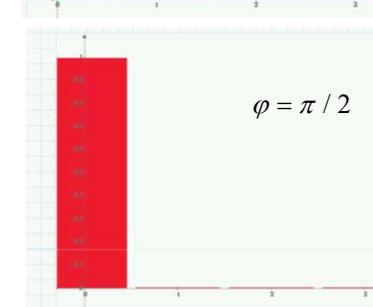
Les modes cohérents
se comportent comme
des rayons laser classiques
??? Pourquoi ???



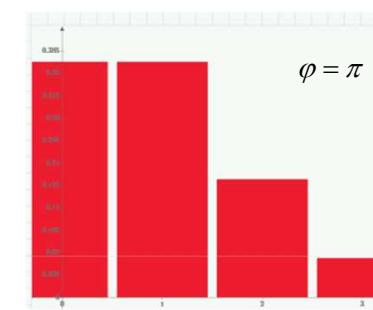
Interférences



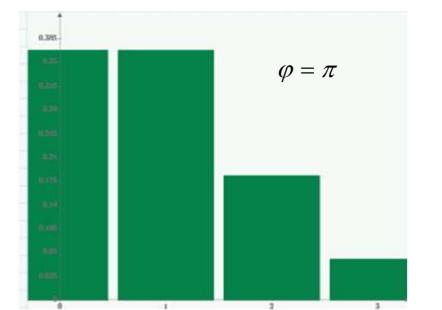
$\varphi = \pi/2$



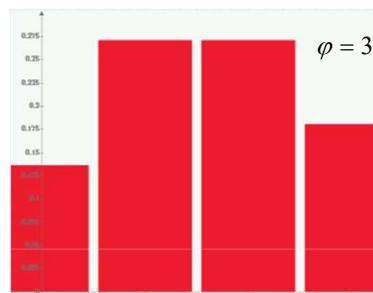
$\varphi = \pi/2$



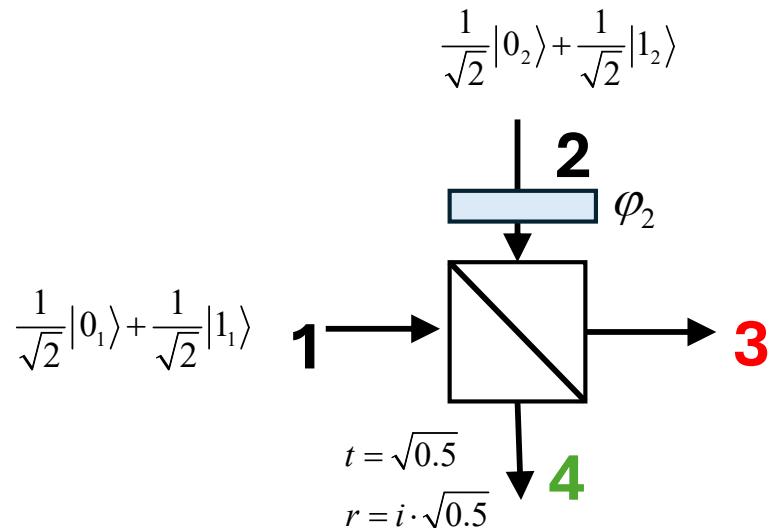
$\varphi = \pi$



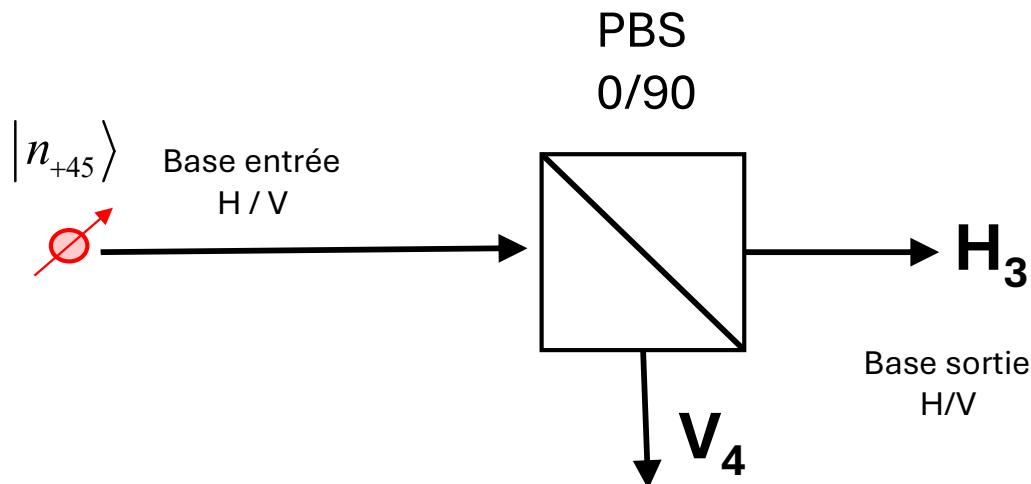
$\varphi = \pi$



$\varphi = 3\pi/2$



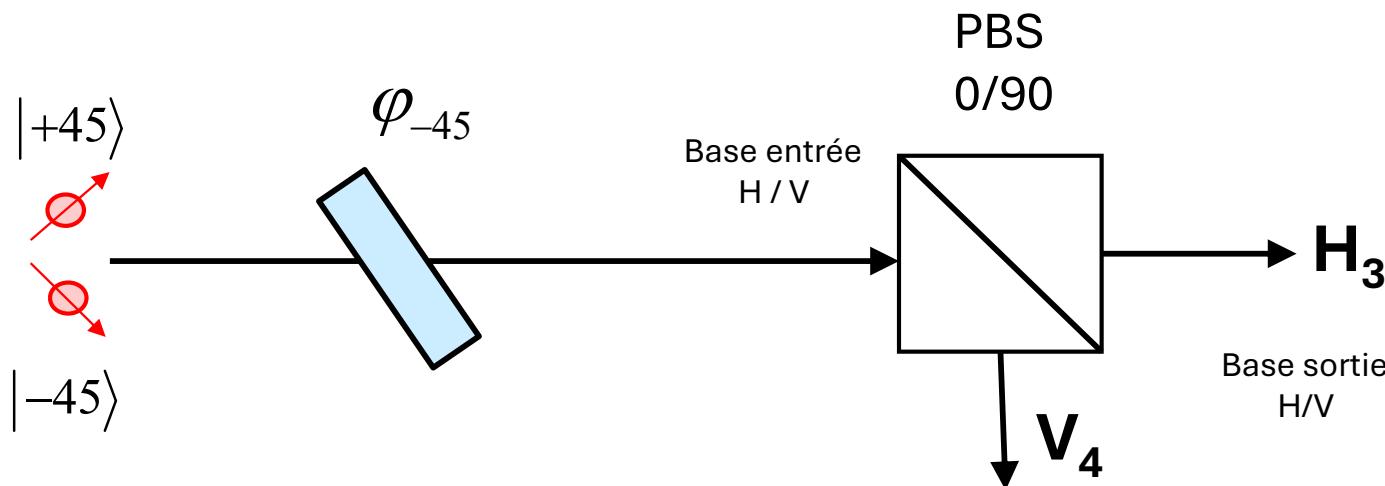
- 1) Calculez le mode à l'entrée
- 2) Interprétez les différentes composantes et déterminez lesquelles donnent lieu à une interférence
- 3) Calculez les modes en sortie et déterminez les nombres de photons en sortie (N3 et N4)



$$|H\rangle \rightarrow 1 \cdot |1_3\rangle$$

$$|V\rangle \rightarrow i \cdot |1_4\rangle$$

- 1) Calculez le mode superposé en sortie si un seul photon polarisé +45 est injecté en entrée (n=1)
- 2) Calculez le mode superposé en sortie si deux photons polarisés +45 sont injectés en entrée (n=2)



$$|H\rangle \rightarrow 1 \cdot |1_3\rangle$$

$$|V\rangle \rightarrow i \cdot |1_4\rangle$$

Un lame biréfringente est introduite avec un axe incliné à -45^0 .

- 1) Calculez le mode superposé en sortie si un photon polarisé $+45$ et un photon polarisé -45 sont injectés simultanément en entrée ($n=1$) en fonction de la phase φ_{-45} introduite par la lame.
- 2) qu'attendez-vous pour le cas où un mode cohérent polarisé $+45$ et un mode cohérent -45 sont injectés avec le même nombre moyen de photons?